

# In-person session 5

**February 9, 2023**

PMAP 8521: Program evaluation  
Andrew Young School of Policy Studies

# Plan for today

**DAGs**

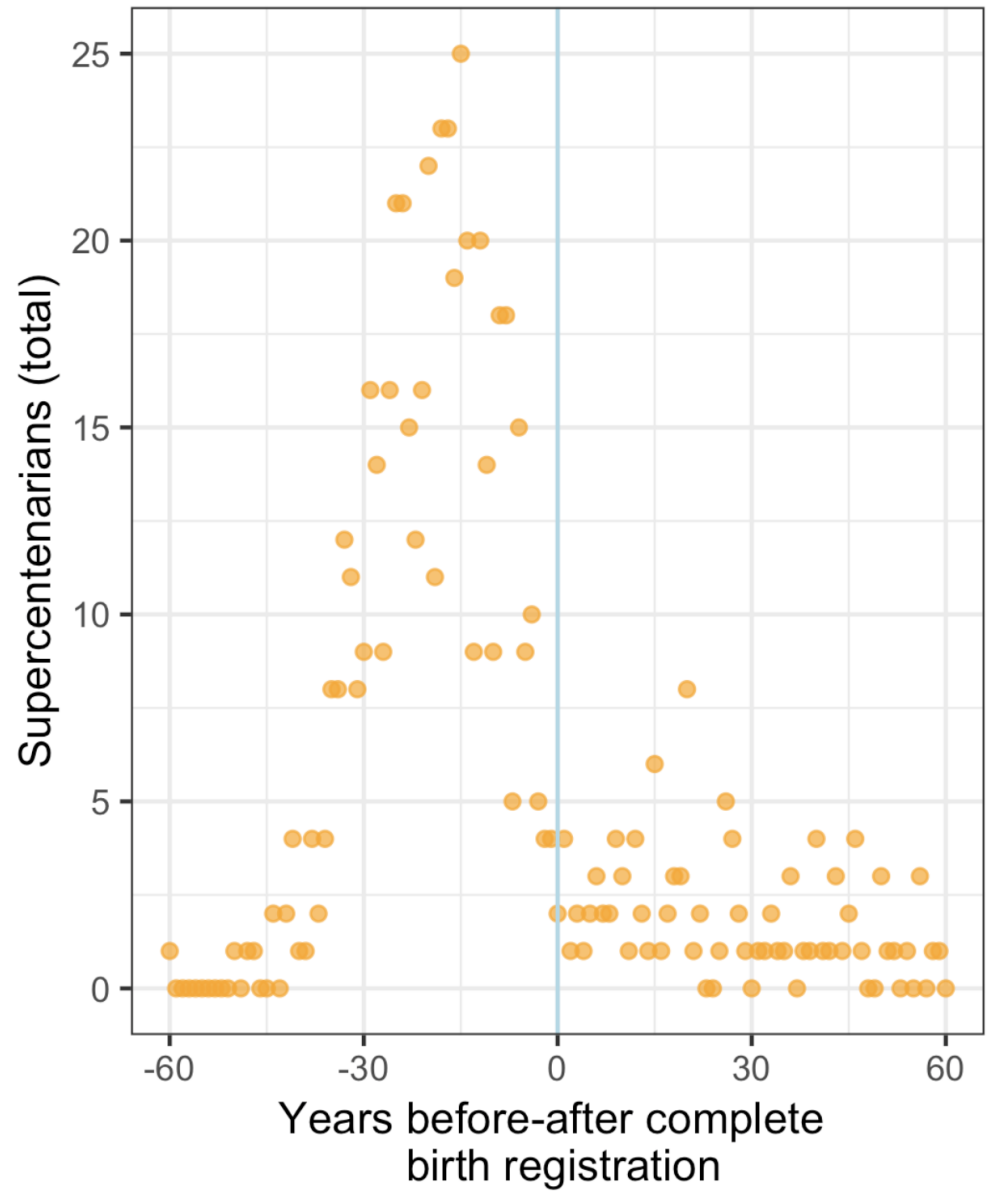
**Logic models, DAGs, and measurement**

**Potential outcomes vs. do() notation**

**do-calculus, adjustment, and CATEs**

# DAGs

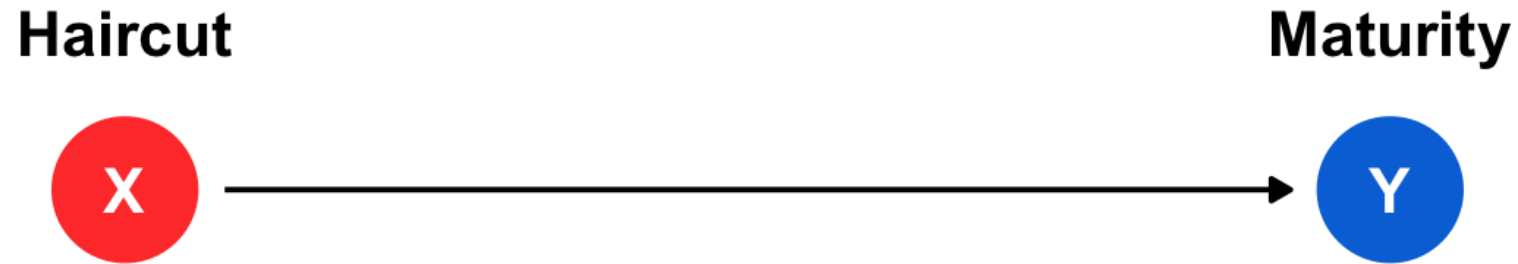
**Causal thinking is necessary—  
even for descriptive work!**



**"Every time I get a haircut, I become more mature!"**

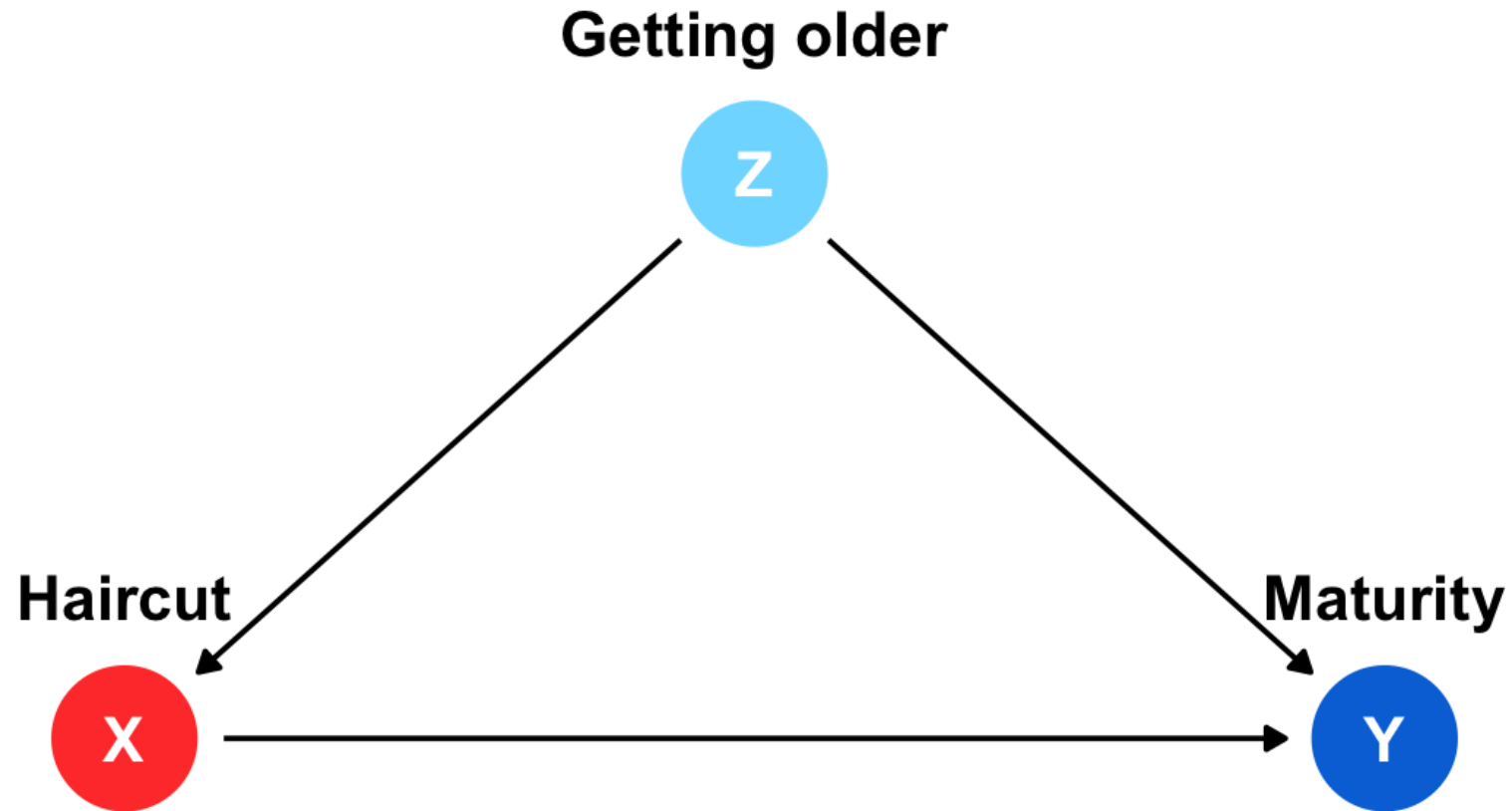


"Every time I get a haircut, I become more mature!"



$$E[\text{Maturity} \mid \text{do}(\text{Get haircut})]$$

# Getting older opens a backdoor path





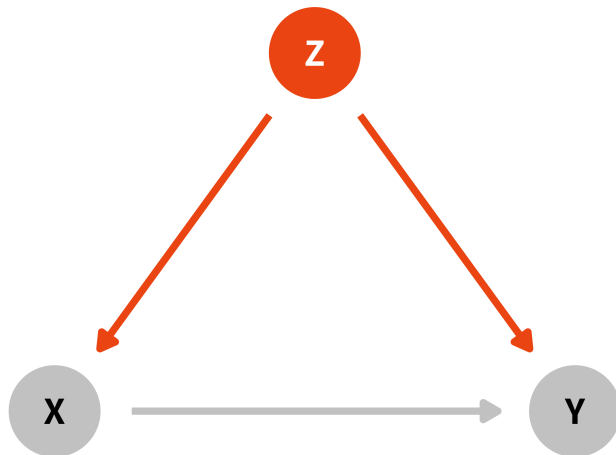
**But what does that mean,  
"opening a backdoor path"?**

**How does statistical association  
get passed through paths?**

# How do I know which of these is which?

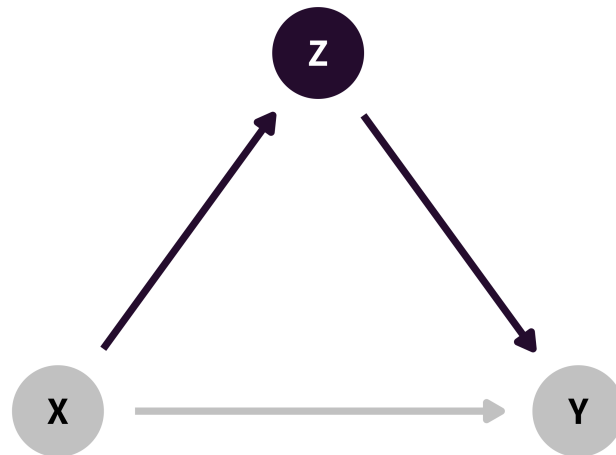
## Confounder

(Fork)



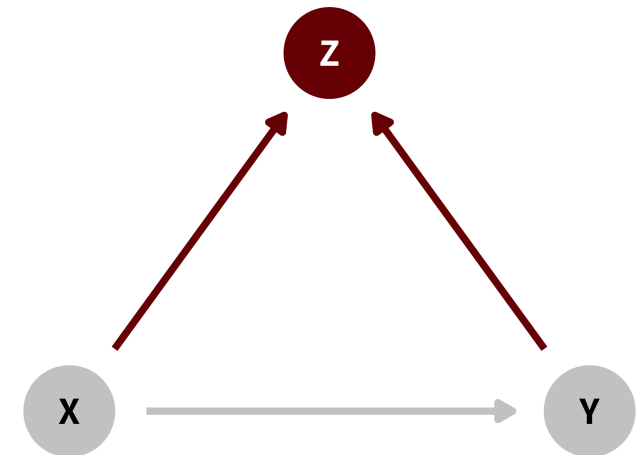
## Mediator

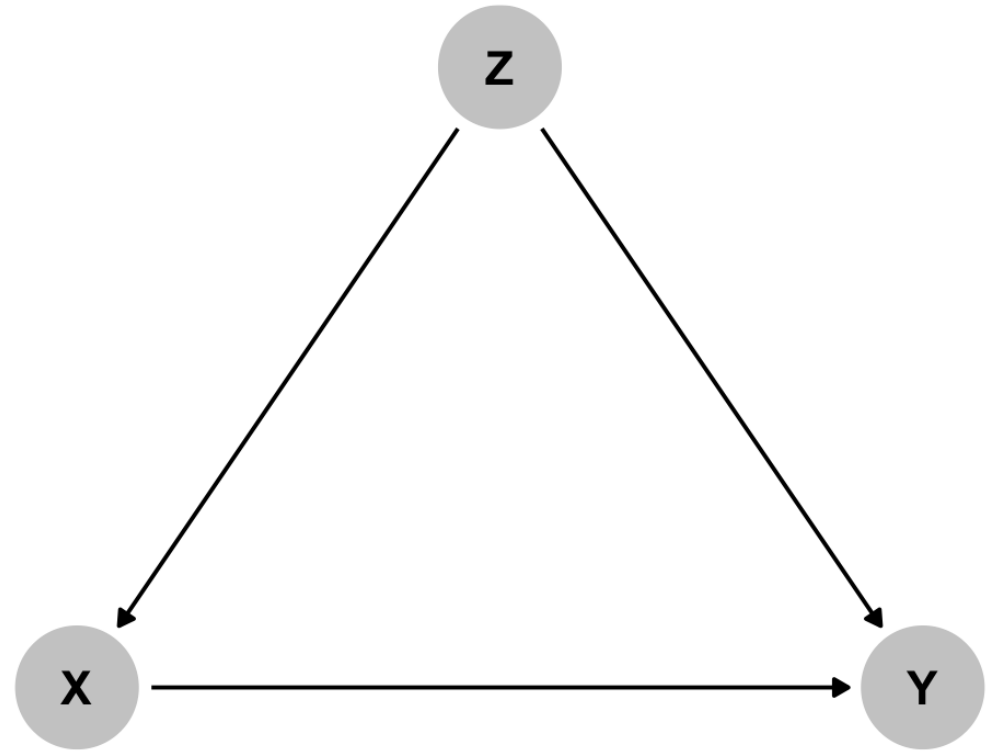
(Chain)

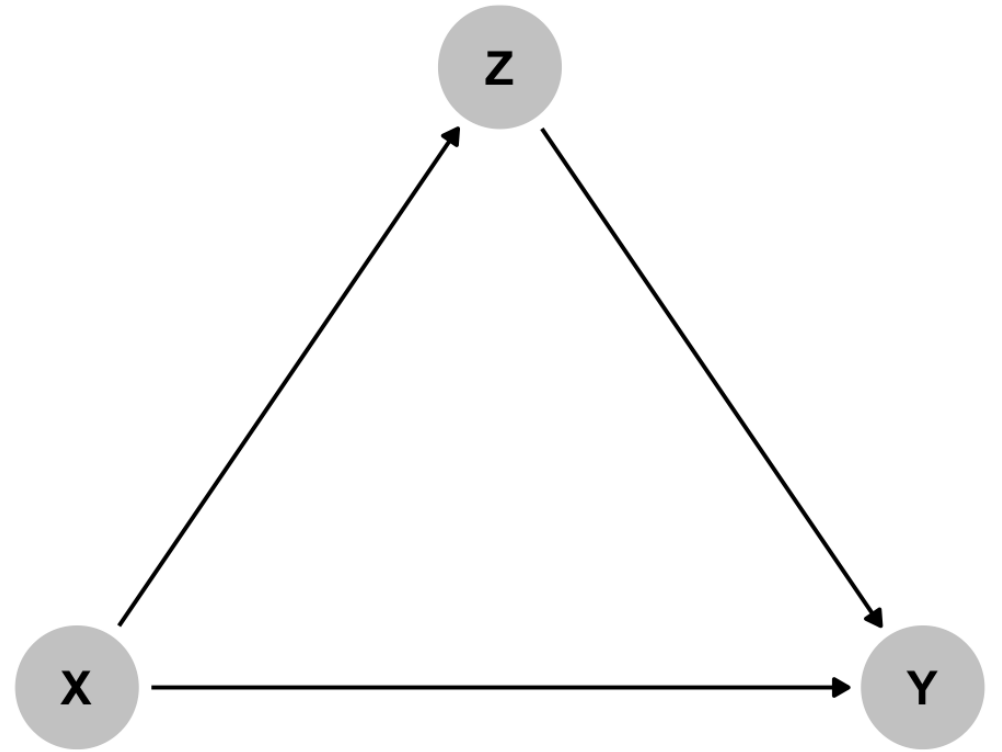


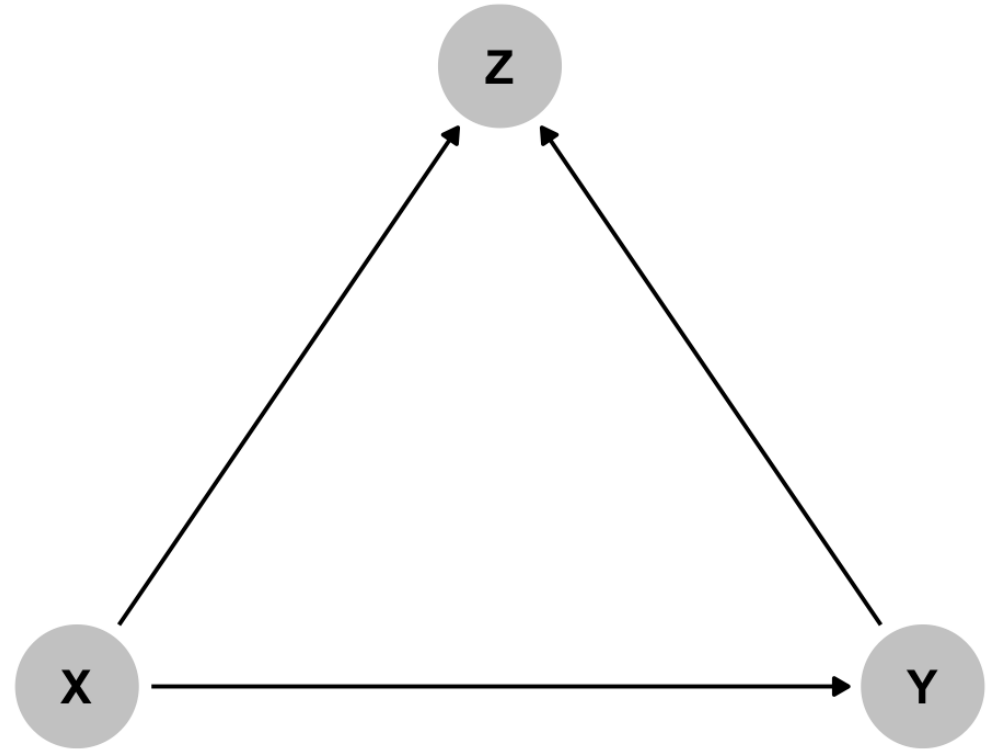
## Collider

(Inverted fork)

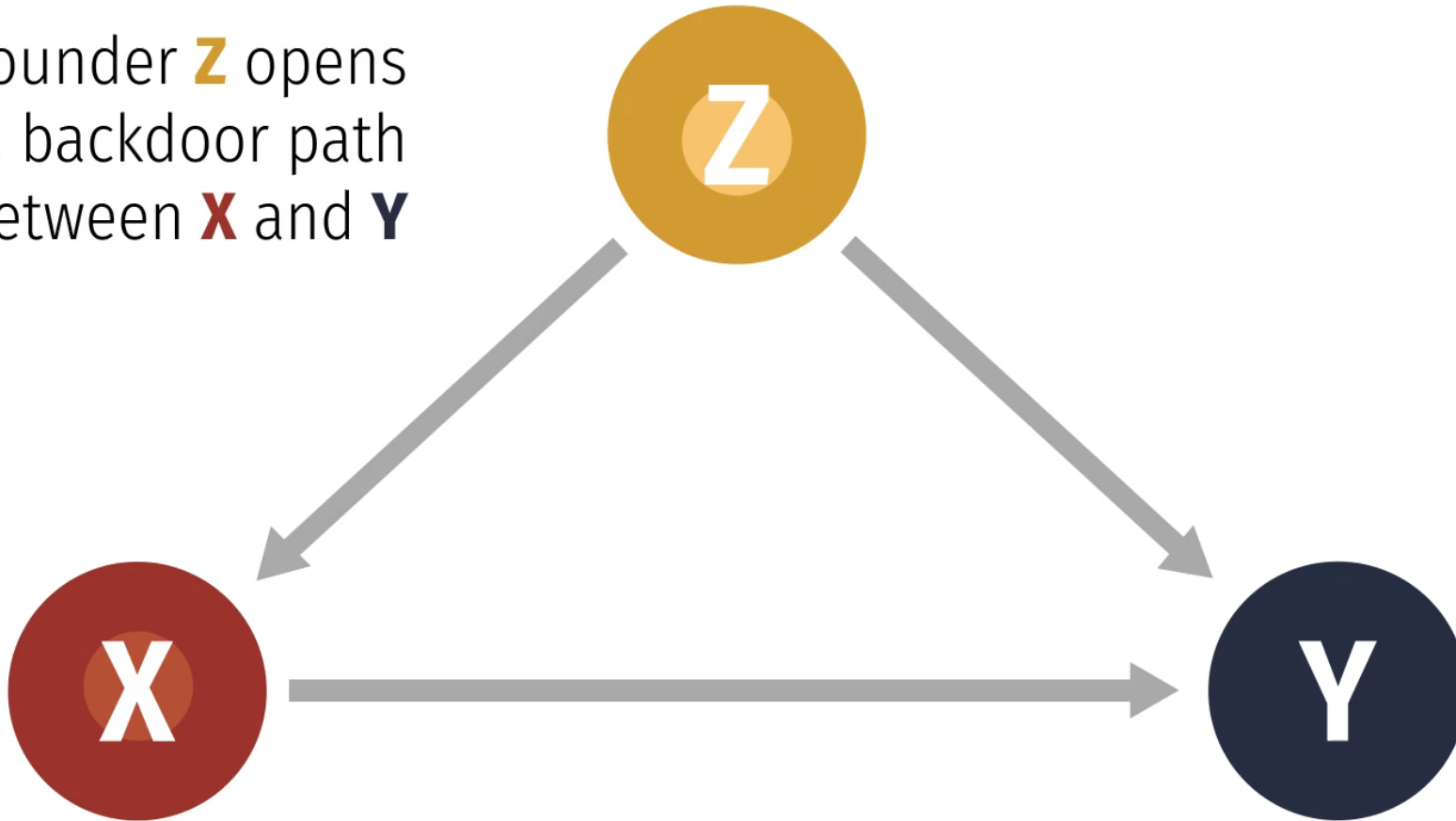








Confounder **Z** opens  
a backdoor path  
between **X** and **Y**

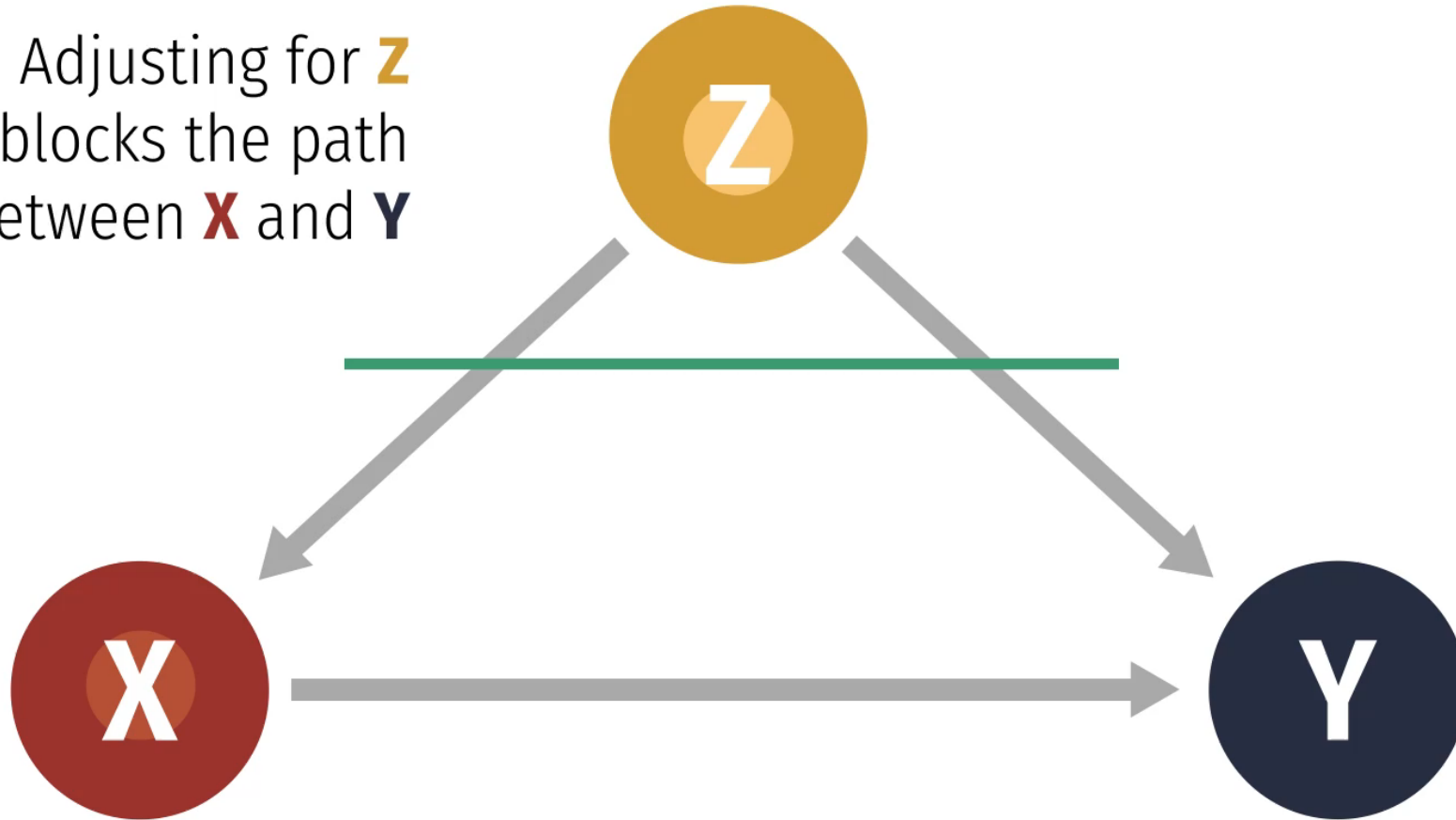


**Y** ~

▶ 0:00 / 0:06

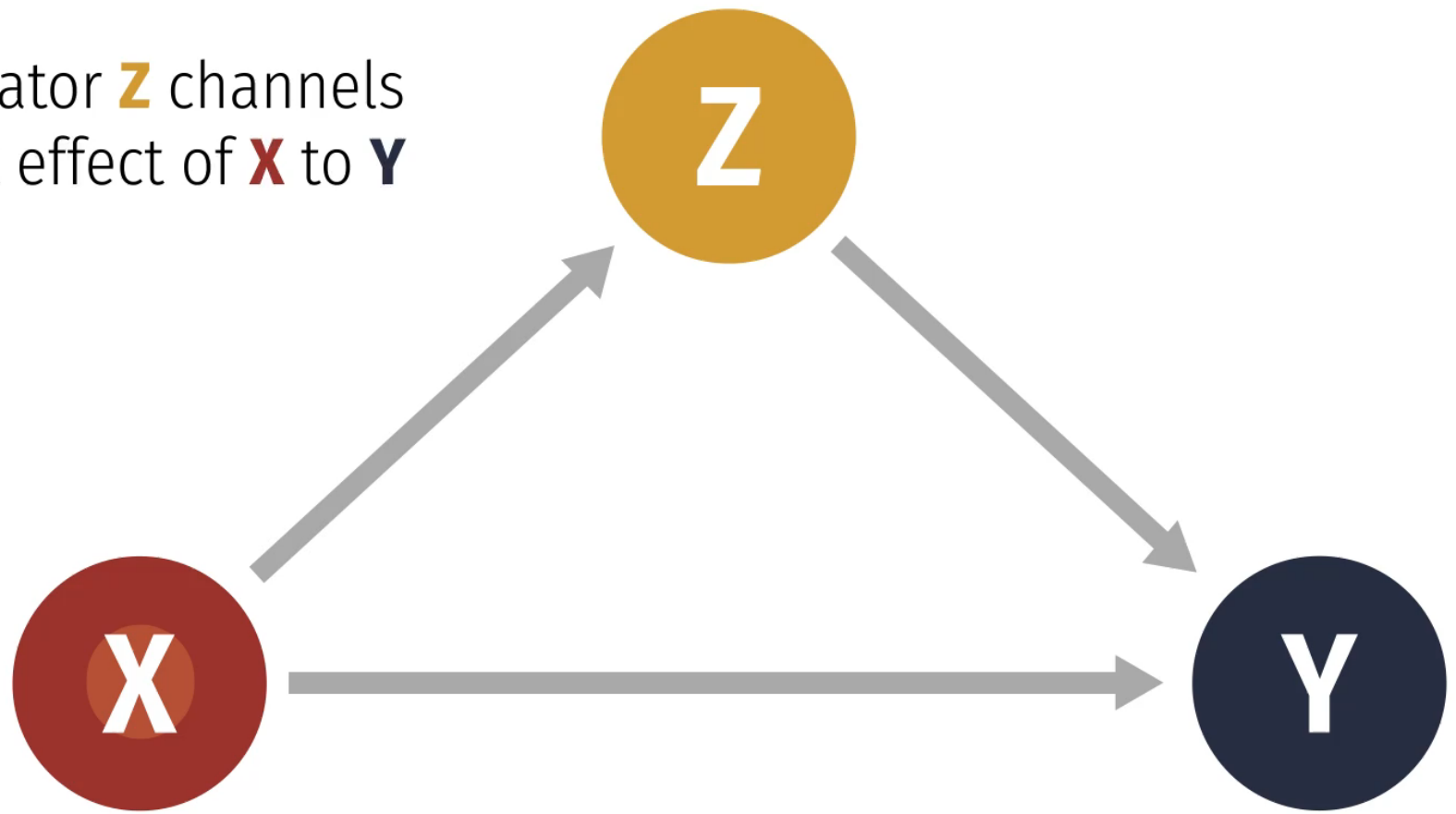


Adjusting for **Z**  
blocks the path  
between **X** and **Y**



**Y** ~

Mediator **Z** channels indirect effect of **X** to **Y**



**Y** ~

0:00 / 0:06

▶ 🔊 🗄️ ⋮

Progress bar



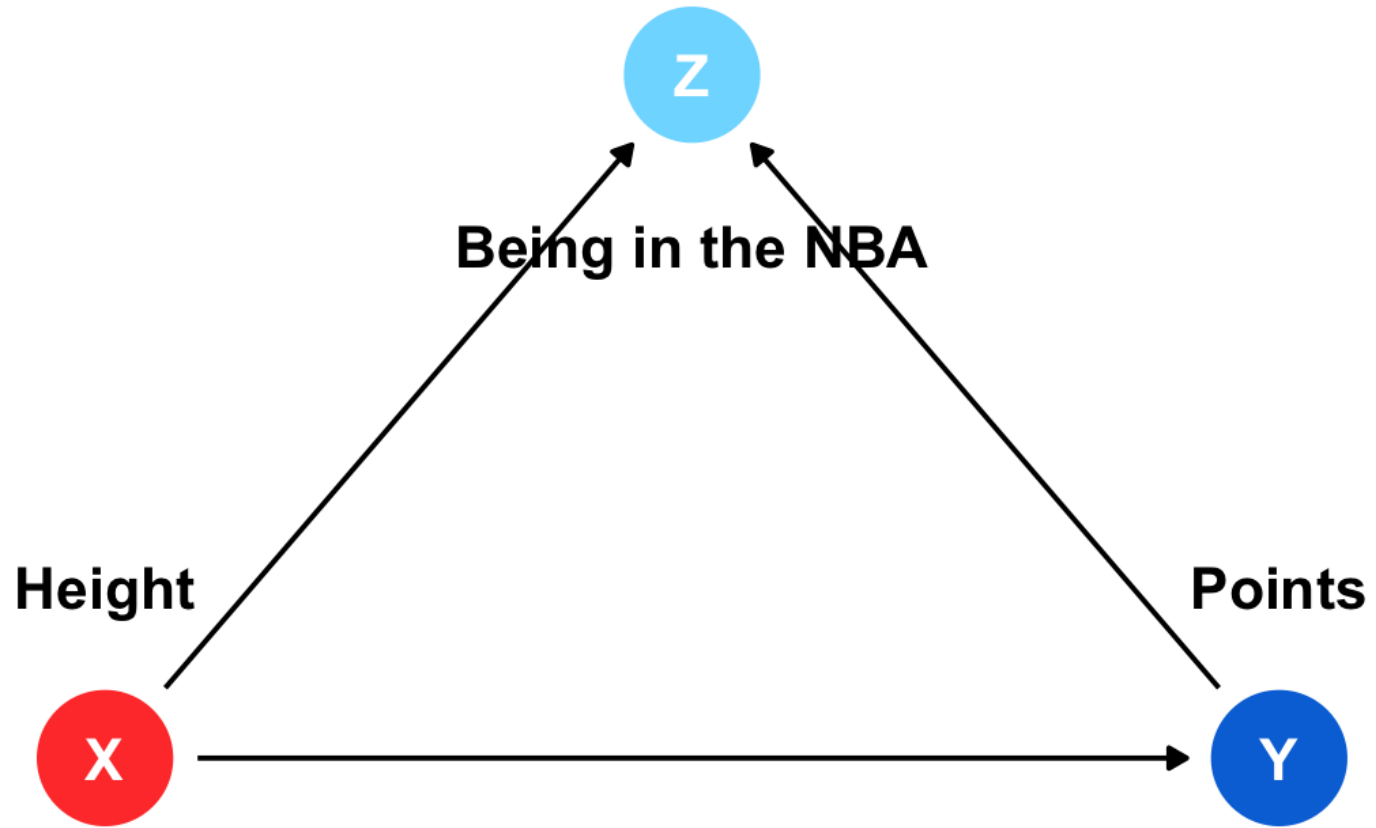
# d-separation

Except for the one arrow between X and Y,  
no statistical association can flow between X and Y

**This is identification—**  
all alternative stories are ruled out  
and the relationship is isolated

**How exactly do colliders  
mess up your results?**

**It looks like you can  
still get the effect of X on Y**





Sept. 10, 2021, 3:58 p.m. ET

By [Davey Alba](#)

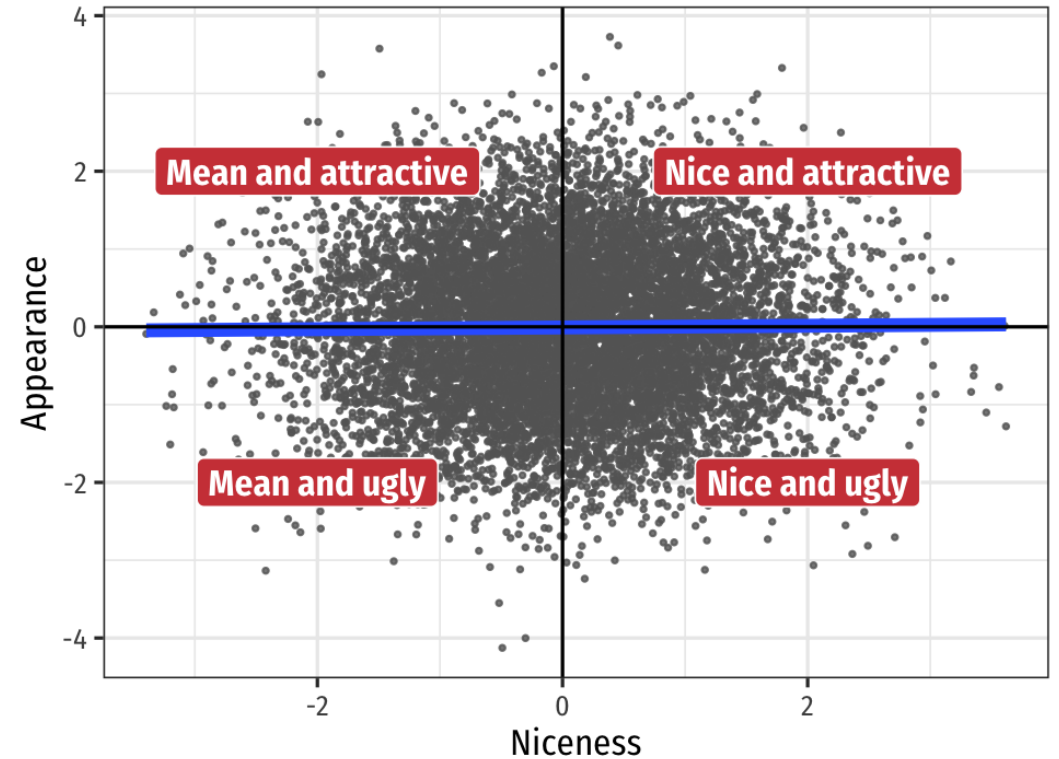
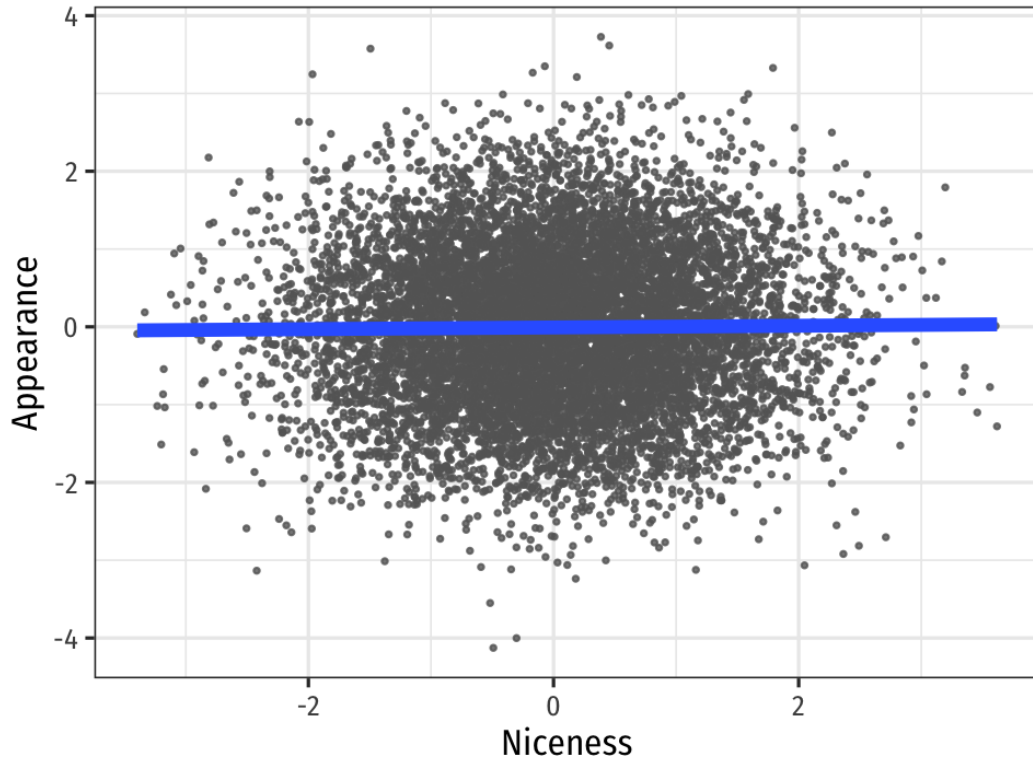


## Facebook sent flawed data to misinformation researchers.

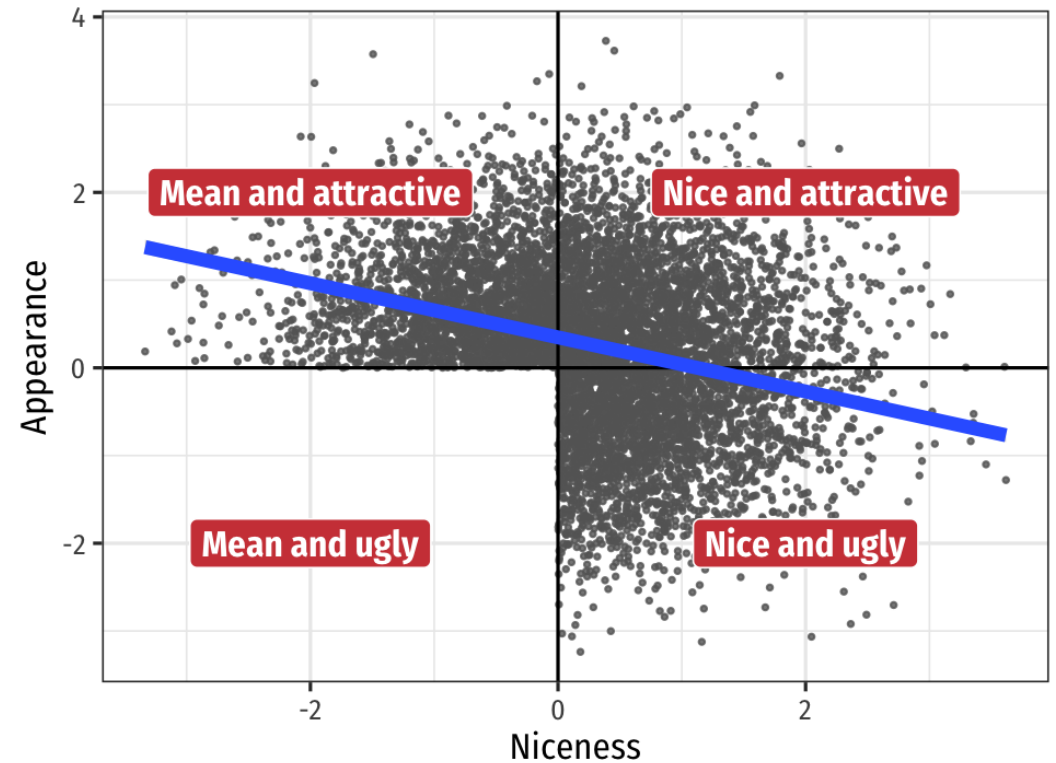
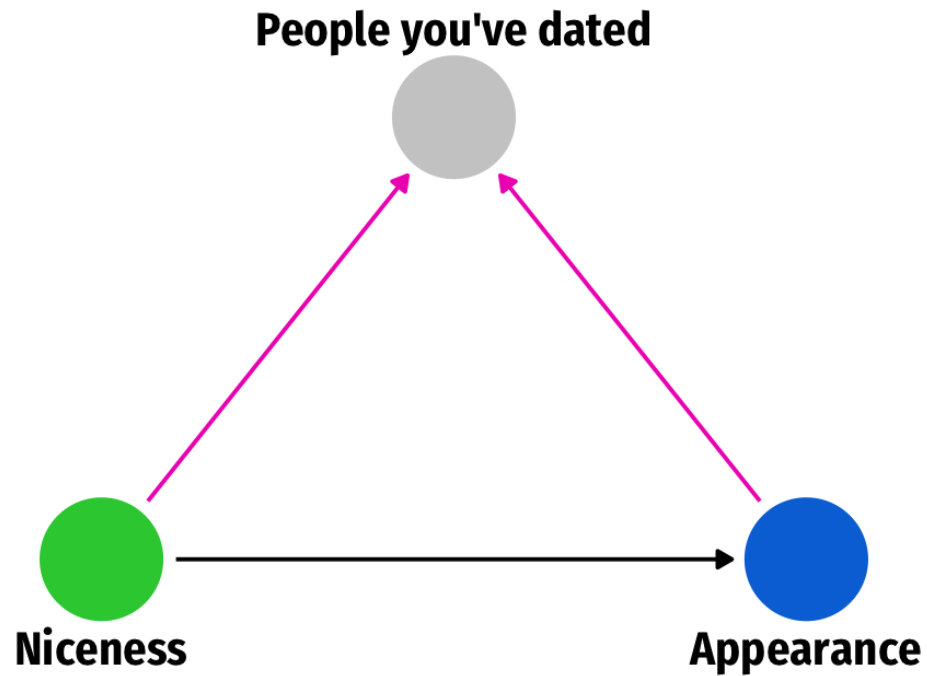


Mark Zuckerberg, chief executive of Facebook, testifying in Washington in 2018. Tom Brenner/The New York Times

# Does niceness improve appearance?

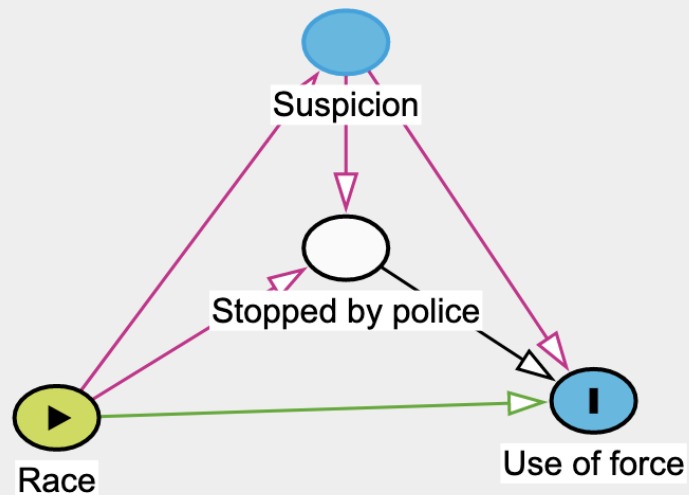


# Collider distorts the true effect!



# Effect of race on police use of force using administrative data

# Effect of race on police use of force using administrative data



American Political Science Review, Page 1 of 19  
doi:10.1017/S0003055420000039

© American Political Science Association 2020

## Administrative Records Mask Racially Biased Policing

DEAN KNOX *Princeton University*

WILL LOWE *Hertie School of Governance*

JONATHAN MUMMOLO *Princeton University*

**R**esearchers often lack the necessary data to credibly estimate racial discrimination in policing. In particular, police administrative records lack information on civilians police observe but do not investigate. In this article, we show that if police racially discriminate when choosing whom to investigate, analyses using administrative records to estimate racial discrimination in police behavior are statistically biased, and many quantities of interest are unidentified—even among investigated individuals—absent strong and untestable assumptions. Using principal stratification in a causal mediation framework, we derive the exact form of the statistical bias that results from traditional estimation. We develop a bias-correction procedure and nonparametric sharp bounds for race effects, replicate published findings, and show the traditional estimator can severely underestimate levels of racially biased policing or mask discrimination entirely. We conclude by outlining a general and feasible design for future studies that is robust to this inferential snare.

Concern over racial bias in policing, and the public availability of large administrative data sets documenting police-civilian interactions, have prompted a raft of studies attempting to quantify the effect of civilian race on law enforcement behavior. These studies consider a range of outcomes including ticketing, stop duration, searches, and the use of force (e.g., Antonovics and Knight 2009; Fryer 2019; Ridgeway 2006; Nix et al. 2017). Most research in this area attempts to adjust for omitted variables that may correlate with suspect race and the outcome of interest. In contrast, this study addresses a more fundamental problem that remains even if the vexing issue of omitted variable bias is solved: the inevitable statistical bias that results from studying racial discrimination using records that are themselves the product of racial discrimination (Angrist and Pischke 2008; Elwert and Winship 2014; Rosenbaum 1984). We show that when there is any

biased absent additional data and/or strong and untestable assumptions.

This study makes several contributions. We clarify the causal estimands of interest in the study of racially discriminatory policing—quantities that many studies appear to be targeting, but are rarely made explicit—and show that the conventional approach fails to recover any known causal quantity in reasonable settings. Next, we highlight implicit and highly implausible assumptions in prior work and derive the statistical bias when they are violated. We proceed to develop informative nonparametric sharp bounds for the range of possible race effects, apply these in a reanalysis and extension of a prominent article on police use of force (Fryer 2019), and present bias-corrected results that suggest this and similar studies drastically underestimate the level of racial bias in police-civilian interactions. Finally, we outline strategies for future data collection and re-



**Smoking → Cardiac arrest example**

**How do you know  
if the DAG is right??**

**How can you be sure  
you include everything in a DAG?**

**How do you know when to stop?**

**Is there a rule of thumb  
for the number of nodes?**

**Why can we combine nodes in a DAG if they don't represent the same concept?**

**Why include unmeasurable things in a DAG?**

**Why do DAGs have to be acyclic?**

**What if there really is reverse causation?**

**How do we actually  
adjust for these things?**

# Logic models, DAGs, and measurement

# What's the difference between logic models and DAGs?

Can't I just remake my logic model in Dagitty and be done?

# DAGs vs. Logic models

**DAGs are a *statistical* tool**

Describe a data-generating process  
and isolate/identify relationships

**Logic models are a *managerial* tool**

Oversee the inner workings of a program and its theory



# Berkeley Will Fully Close Its Streets to Create Giant Outdoor Dining Rooms

3

Berkeley is moving fast to expand outdoor dining

by [Eve Batey](#) | May 14, 2020, 1:02pm PDT

[f](#) [t](#) [SHARE](#)



**E**  
**Eater SF**

Sign up for our newsletter.

Email (required)

By signing up, you agree to our [Privacy Notice](#) and European users agree to the data transfer policy.

**SUBSCRIBE**






Cities can prepare for climate change emergencies by adding green spaces to help manage stormwater, heat stress and air quality. (Shutterstock)

- Email
- Twitter 88
- Facebook 1.6k
- LinkedIn
- Print

The COVID-19 pandemic has forced governments to weigh the benefits of keeping green spaces open against the public health concerns that come from their use. During the pandemic, playgrounds have been taped off, parks locked and access to outdoor spaces for recreation cut off.

Green spaces have positive effects on mental health, physical fitness, social cohesion and spiritual wellness. Although researchers say the coronavirus spreads more easily indoors than outdoors, they also believe the concentrated use of green spaces will increase the transmission of COVID-19.

#### Authors

-  **Ryan Plummer**  
Professor, Environmental Sustainability Research Centre, Brock University
-  **Darby McGrath**  
Adjunct professor, Environmental Sustainability Research Centre, Brock University
-  **Sivajani Sivarajah**  
Research Associate, Department of Architectural Science, Ryerson University

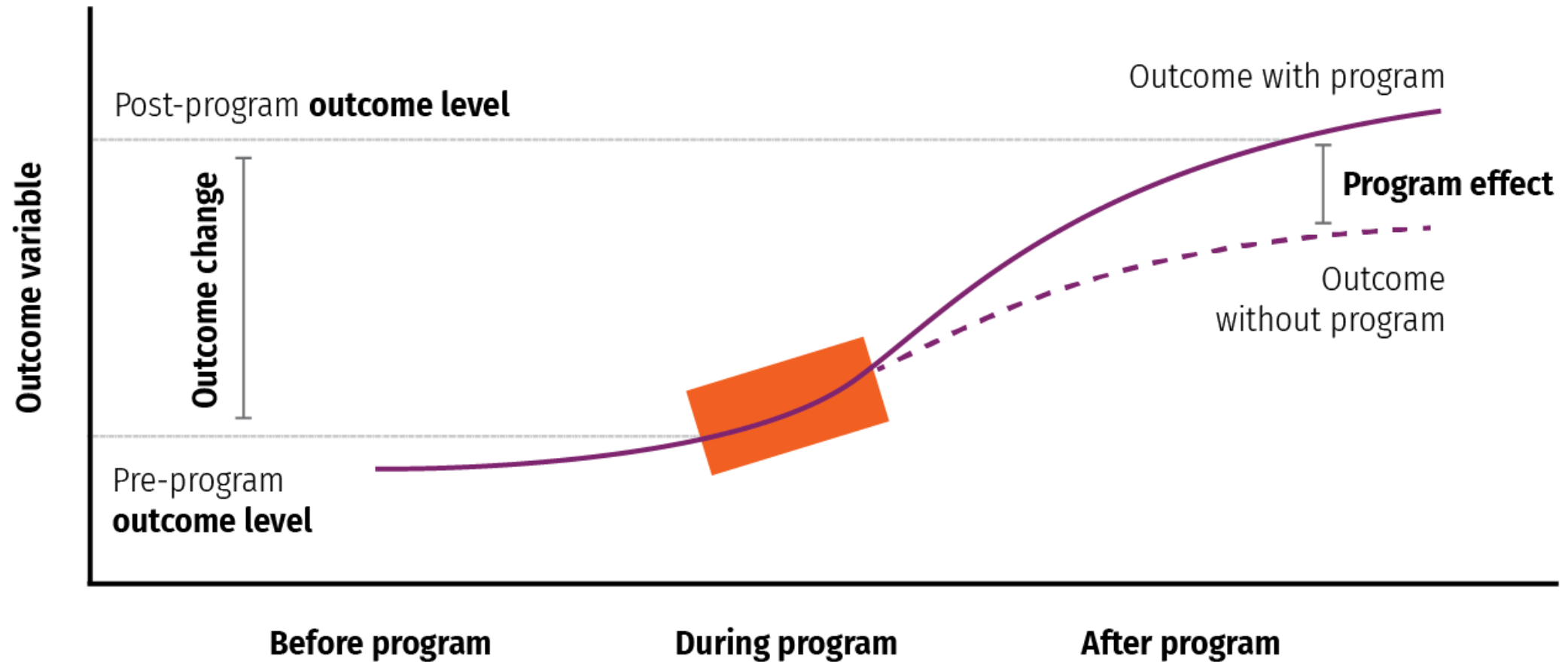
# Potential outcomes vs. `do()` notation

# Expectations

$\mathbf{E}(\cdot)$ ,  $\mathbf{E}(\cdot)$ ,  $\mathbb{E}(\cdot)$  vs.  $\mathbf{P}(\cdot)$

Basically a fancy way of saying "average"

# Outcomes and programs



# Causal effects with potential outcomes

**Potential outcomes notation:**

$$\delta = \frac{1}{n} \sum_{i=1}^n Y_i(1) - Y_i(0)$$

or alternatively with **E**

$$\delta = \mathbf{E}[Y_i(1) - Y_i(0)]$$

# Causal effects with do()

**Pearl notation:**

$$\delta = \mathbf{E}[Y_i \mid \text{do}(X = 1) - Y_i \mid \text{do}(X = 0)]$$

or more simply

$$\delta = \mathbf{E}[Y_i \mid \text{do}(X)]$$

$$\mathbf{E}[Y_i \mid \text{do}(X)] \\ = \\ \mathbf{E}[Y_i(1) - Y_i(0)]$$

**We can't see this**

$$\mathbf{E}[Y_i \mid \text{do}(X)] \quad \text{or} \quad \mathbf{E}[Y_i(1) - Y_i(0)]$$

**So we find the average causal effect (ACE)**

$$\hat{\delta} = \mathbf{E}[Y_i \mid X = 1] - \mathbf{E}[Y_i \mid X = 0]$$



The average  
population-level  
change in  $y$  when  
*directly intervening*  
(or doing)  $x$

$$\mathbf{E}(y \mid \mathbf{do}(x))$$

Causation

The average  
population-level  
change in  $y$  when  
accounting for  
*observed*  $x$

$$\mathbf{E}(y \mid x)$$

Correlation

$\neq$

# do-calculus, adjustment, and CATEs

# DAGs and identification

DAGs are a statistical tool, but they don't tell you what statistical method to use

DAGs help you with the **identification strategy**



**Thomas Massie** ✓

@RepThomasMassie



Over 70% of Americans who died with COVID, died on Medicare, and some people want [#MedicareForAll](#) ?

11:00 AM · Feb 9, 2022 · Twitter for iPhone

# Easist identification

Identification through research design

RCTs

When treatment is randomized, delete all arrows going into it

No need for any do-calculus!

# Most other identification

Identification through do-calculus

Rules for graph surgery

Backdoor adjustment and frontdoor adjustment  
are special common patterns of do-calculus

# Where can we learn more about *do*-calculus?

Here!

**The do-calculus** Let  $G$  be a CGM,  $G_{\overline{T}}$  represent  $G$  post-intervention (i.e with all links into  $T$  removed) and  $G_{\underline{T}}$  represent  $G$  with all links out of  $T$  removed. Let  $do(t)$  represent intervening to set a single variable  $T$  to  $t$ ,

**Rule 1:**  $\mathbb{P}(y|do(t), z, w) = \mathbb{P}(y|do(t), z)$  if  $Y \perp\!\!\!\perp W|(Z, T)$  in  $G_{\overline{T}}$

**Rule 2:**  $\mathbb{P}(y|do(t), z) = \mathbb{P}(y|t, z)$  if  $Y \perp\!\!\!\perp T|Z$  in  $G_{\underline{T}}$

**Rule 3:**  $\mathbb{P}(y|do(t), z) = \mathbb{P}(y|z)$  if  $Y \perp\!\!\!\perp T|Z$  in  $G_{\overline{T}}$ ,

**Supplement 2. The do-calculus**

The *do*-calculus is an axiomatic system for replacing probability formulas involving the *do* operator with ordinary conditional probabilities. It consists of three axiom schemas that have graphical criteria for when certain substitutions may be made.

Where  $G$  is the ADMG on variable set  $V$ , and  $P$  satisfies (MC -  $d$ -separation), the rules are:

**Rule 1 (Insertion/deletion of observations)**

$$P(Y | do(X), Z, W) = P(Y | do(X), W)$$

if  $Y$  and  $Z$  are  $d$ -separated by  $X \cup W$  in  $G^*$ , where  $G^*$  is the graph obtained from  $G$  by removing all arrows pointing into variables in  $X$ .

**Rule 2 (Action/observation exchange)**

$$P(Y | do(X), do(Z), W) = P(Y | do(X), Z, W)$$

if  $Y$  and  $Z$  are  $d$ -separated by  $X \cup W$  in  $G$  and  $Z$  is not pointing out of variables in  $Z$ .

**Rule 3 (Insertion/deletion of actions)**

$$P(Y | do(X), do(Z), W) = P(Y | do(X), W)$$

if  $Y \perp\!\!\!\perp Z | T, W$  in  $G_{\overline{T}}$  where  $T$  is the set of nodes of  $Z$  that aren't ancestors of any node in  $W$ .

**Theorem 6.2 (Rules of do-calculus)** Given a causal graph  $G$ , an associated distribution  $P$ , and disjoint sets of variables  $Y, T, Z$ , and  $W$ , the following rules hold.

**Rule 1:**

$$P(y | do(t), z, w) = P(y | do(t), w) \quad \text{if } Y \perp\!\!\!\perp_{G_{\overline{T}}} Z | T, W \quad (6.18)$$

**Rule 2:**

$$P(y | do(t), do(z), w) = P(y | do(t), z, w) \quad \text{if } Y \perp\!\!\!\perp_{G_{\underline{T}, Z}} Z | T, W \quad (6.19)$$

**Rule 3:**

$$P(y | do(t), do(z), w) = P(y | do(t), w) \quad \text{if } Y \perp\!\!\!\perp_{G_{\overline{T}, Z(W)}} Z | T, W \quad (6.20)$$

**Rule 1:** Decide if we can ignore an observation

$$P(y \mid z, \text{do}(x), w) = P(y \mid \text{do}(x), w) \quad \text{if } (Y \perp Z \mid W, X)_{G_{\overline{X}}}$$

**Rule 2:** Decide if we can treat an intervention as an observation

$$P(y \mid \text{do}(z), \text{do}(x), w) = P(y \mid z, \text{do}(x), w) \quad \text{if } (Y \perp Z \mid W, X)_{G_{\overline{X}, \underline{Z}}}$$

**Rule 3:** Decide if we can ignore an intervention

$$P(y \mid \text{do}(z), \text{do}(x), w) = P(y \mid \text{do}(x), w) \quad \text{if } (Y \perp Z \mid W, X)_{G_{\overline{X}, \overline{Z(W)}}}$$



[Marginalization across  $z$  + chain rule for conditional probabilities]

$$P(y \mid \text{do}(x)) = \sum_z P(y \mid \text{do}(x), z) \times P(z \mid \text{do}(x))$$

[Use Rule 2 to treat  $\text{do}(x)$  as  $x$ ]

$$= \sum_z P(y \mid x, z) \times P(z \mid \text{do}(x))$$

[Use Rule 3 to nuke  $\text{do}(x)$ ]

$$= \sum_z P(y \mid x, z) \times P(z \mid \text{nothing!})$$

[Final backdoor adjustment formula!]

$$= \sum_z P(y \mid x, z) \times P(z)$$

# Adjusting for backdoor confounding

Causal effect  
of  $x$  on  $y$

Conditional  
mean of  $y$ ,  
given  $x$  and  $z$ ...  
... weighted  
by  $z$

$$\mathbf{E}(y \mid \text{do}(x)) = \sum_z \mathbf{E}(y \mid x, z) \times \mathbf{P}(z)$$

Sum across  
all values of  $z$

# Adjusting for frontdoor confounding

**More complex DAGs without  
obvious backdoor or frontdoor solutions**

**Chug through the rules of do-calculus  
to see if the relationship is identifiable**

**Causal Fusion**

Summary

Treatment  $X$   
 Outcome  $Y$   
 Adjusted :  
 Query :  $P_X(Y)$

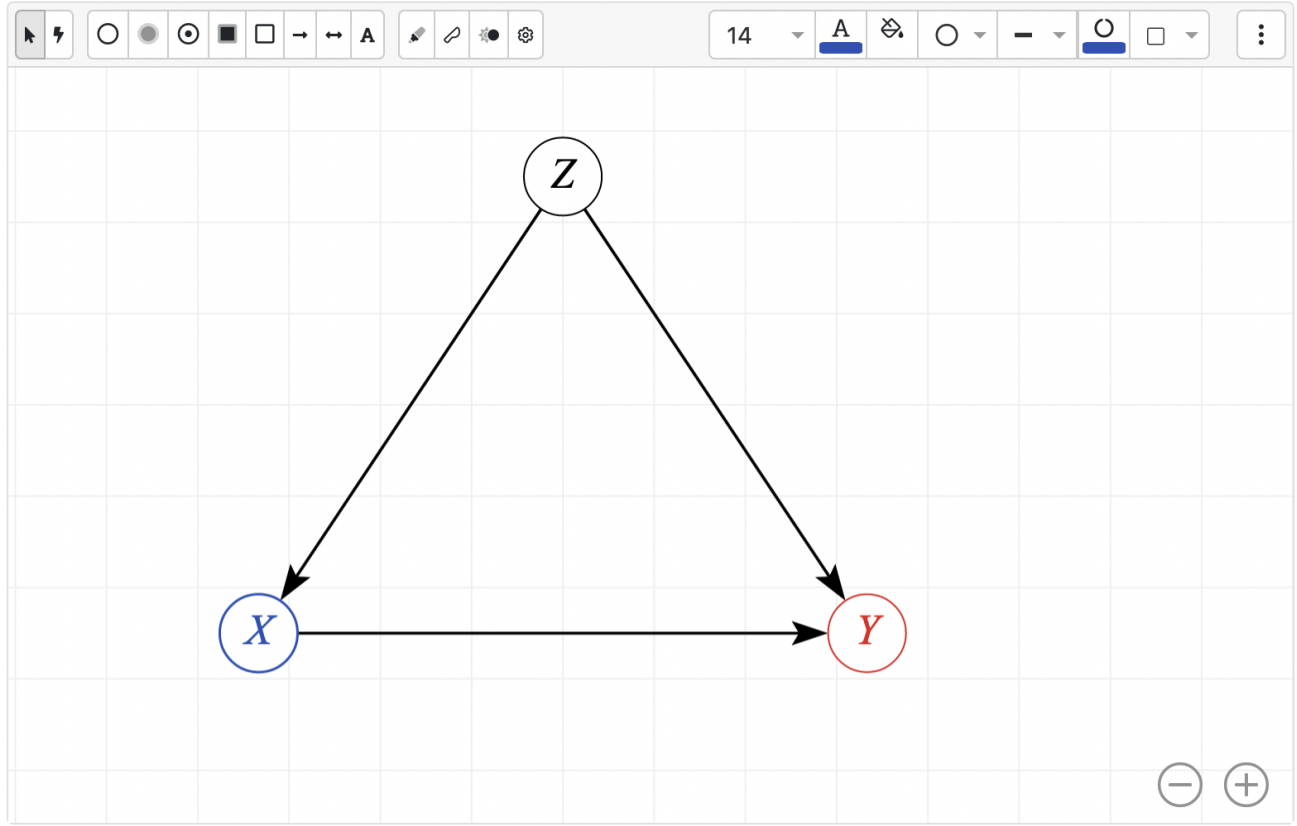
Show More Details

Editor

Graphical Structural

Refresh

- 1 <NODES>
- 2 X -100,75
- 3 Y 100,75
- 4 Z 0,-75
- 5
- 6 <EDGES>
- 7 X → Y
- 8 Z → X
- 9 Z → Y



Confounding Analysis

- Admissible Sets
- Admissibility Test
- Instrumental Variables
- IV Admissibility Test

Path Analysis

- D-Separation
- Causal Paths
- Confounding Paths
- Biasing Paths

Do-Calculus Analysis

- Do-Inspector
- Do-Separation

$\sigma$ -Calculus Analysis

- $\sigma$ -Inspector

Compute

The causal effect of X on Y conditional on with do :

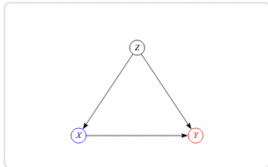
(Query:  $P_X(Y)$  1 Non-Parametric



Clear

1

$$P_X(Y) = \sum_Z P(Y|X, Z) P(Z)$$



- Load
- Estimation
- Derivation
- Remove

**Simplified**

Obtained by Back-Door adjustment with an admissible set {Z}

Do-Calculus

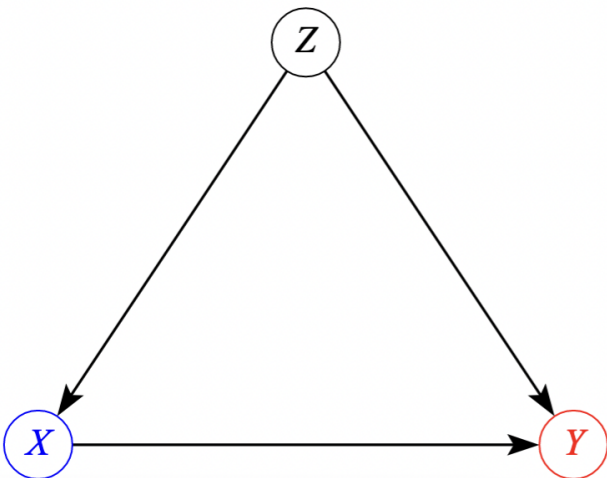
↗ ↘ - + ↻ ⬇

- $P_X(Y)$  (1)
- $\sum_Z P_X(Y|Z) P_X(Z)$  Summing over: Z (2)
- $\sum_Z P(Y|X, Z) P_X(Z)$  Rule 2:  $(X \perp Y|Z)_{G_X}$  (3)
- $\sum_Z P(Y|X, Z) P(Z)$  Rule 3:  $(X \perp Z)_{G_{\bar{X}}}$  (4)

Finally we get:  $\sum_Z P(Y|X, Z) P(Z)$

Subgraph:

Show non-active nodes/edges





Summary

Treatment  $X$

Outcome  $Y$

Adjusted :

Query :  $P_X(Y)$

Show More Details

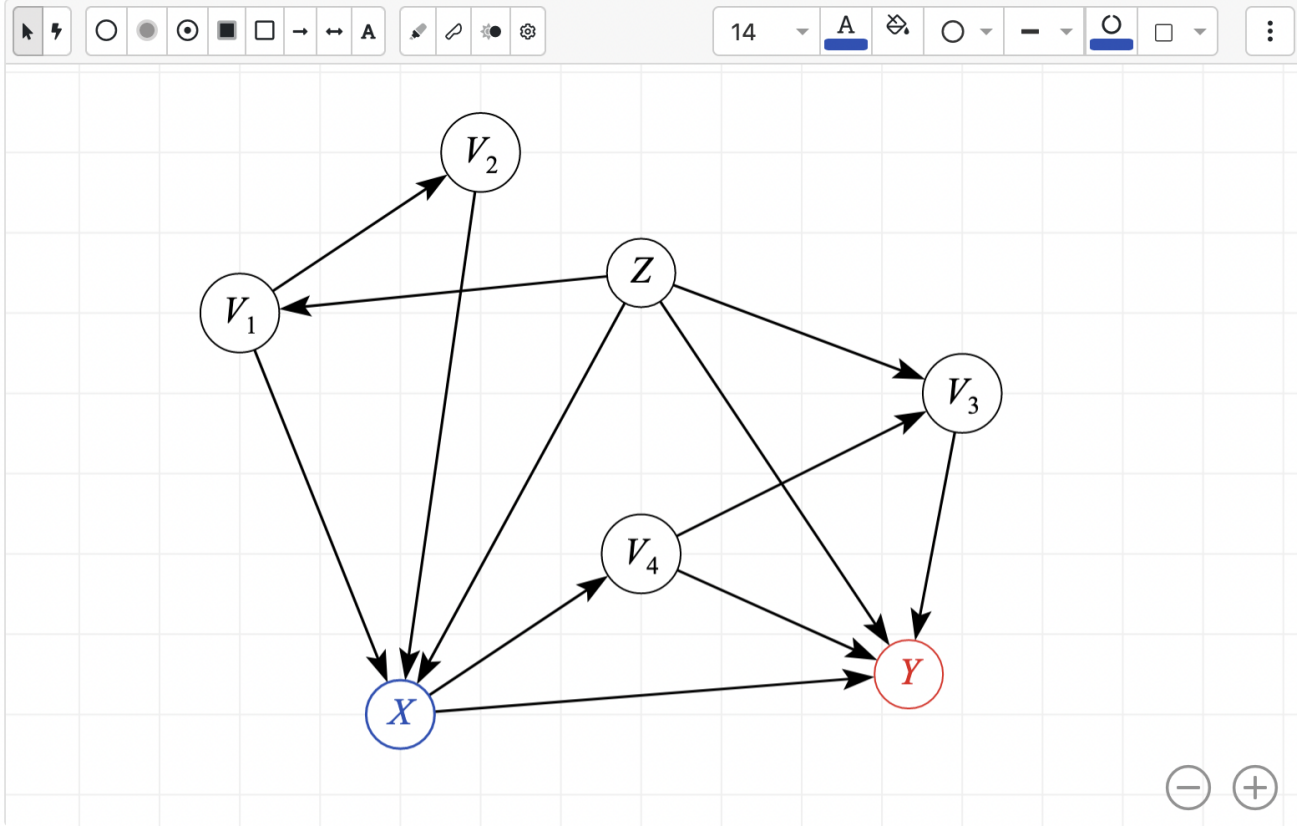
Editor

Graphical

Structural

Refresh

- 1 <NODES>
- 2 X -90, 90
- 3 Y 100, 75
- 4 Z 0, -75
- 5 V\_1 -150, -60
- 6 V\_2 -60, -120
- 7 V\_3 120, -30
- 8 V\_4 0, 30
- 9



Confounding Analysis

Admissible Sets

Admissibility Test

Instrumental Variables

IV Admissibility Test

Path Analysis

D-Separation

Causal Paths

Confounding Paths

Biasing Paths

Do-Calculus Analysis

Do-Inspector

Do-Separation

$\sigma$ -Calculus Analysis

$\sigma$ -Inspector

$\sigma$ -Separation

Compute

The causal effect of

X

on

Y

conditional on

with do :



(Query:  $P_X(Y)$  †

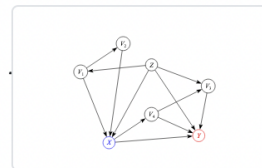
Non-Parametric



Clear

1

$P_X(Y)$  is not identifiable from  $P(V_1, V_2, V_3, V_4, X, Y, Z)$  and  $P_X(V_1, V_2, V_3,$



Load

Remove



**When things are identified, there are  
still arrows leading into Y.  
What do we do with those?  
How do you explain those relationships?**

**Outcomes have multiple causes.  
How do you justify that your proposed  
cause is the most causal factor?**

**Why can't we just subtract the averages between treated and untreated groups?**

**When you're making groups for CATE, how do you decide what groups to put people in?**

Slides from lecture

# Unconfoundedness assumption

How can we assume/pretend that treatment was randomly assigned within each age?

It seems unlikely. Wouldn't there be other factors within the older/younger group that make a person more/less likely to engage in treatment (e.g., health status)?

Slides from lecture

**Does every research question  
need an identification strategy?**

**No!**

**Correlation alone is okay!  
Can lead to more focused causal questions later!**

BREAKING | Jan 14, 2022, 12:34pm EST | 145,393 views

# Moderna Starts Human Trials Of mRNA Vaccine For Virus That Likely Causes Multiple Sclerosis



**Robert Hart** Forbes Staff

Business

*I cover breaking news.*

Follow



Listen to article 3 minutes



**TOPLINE** Moderna recently launched early stage clinical trials for an mRNA vaccine against the Epstein-Barr virus (EBV), a common pathogen that infects almost everyone at some point in their lives, is the primary cause of mononucleosis and, according to a study published in the journal *Science* Thursday, likely causes multiple sclerosis (MS), offering hope the devastating neurological condition might be prevented.