DAGs and potential outcomes

Session 5

PMAP 8521: Program evaluation
Andrew Young School of Policy Studies

Plan for today

do()ing observational causal inference

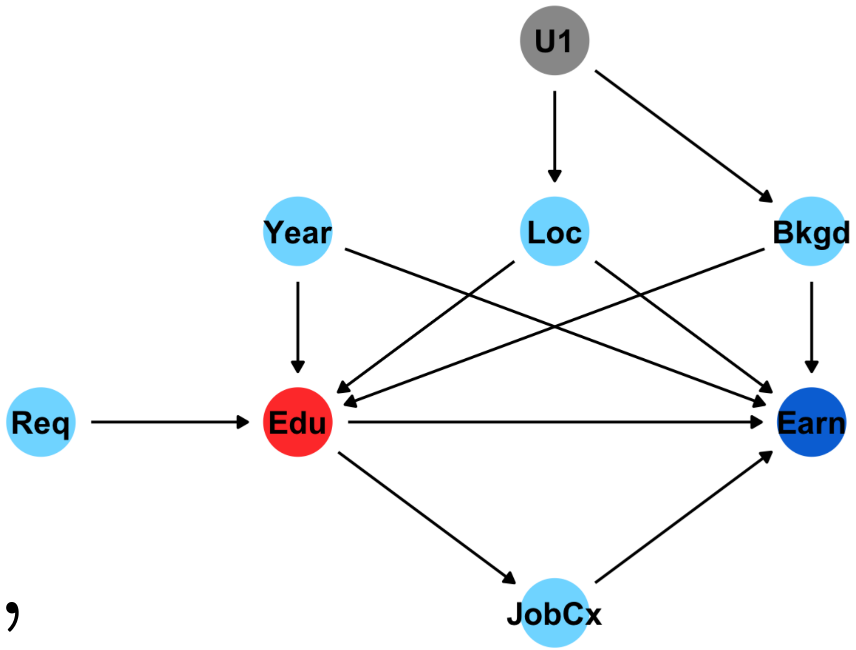
Potential outcomes

do()ing observational causal inference

Structural models

The relationship between nodes can be described with equations

$$
\begin{aligned}
\text{Loc} &= f_{\text{Loc}}(\text{U1}) \\
\text{Bkgd} &= f_{\text{Bkgd}}(\text{U1}) \\
\text{JobCx} &= f_{\text{JobCx}}(\text{Edu}) \\
\text{Edu} &= f_{\text{Edu}}(\text{Req}, \text{Loc}, \text{Year}) \\
\text{Earn} &= f_{\text{Earn}}(\text{Edu}, \text{Year}, \text{Bkgd}, \\
&\qquad \text{Loc}, \text{JobCx})
\end{aligned}
$$



Structural models

`dagify()` in ggdag forces you to think this way

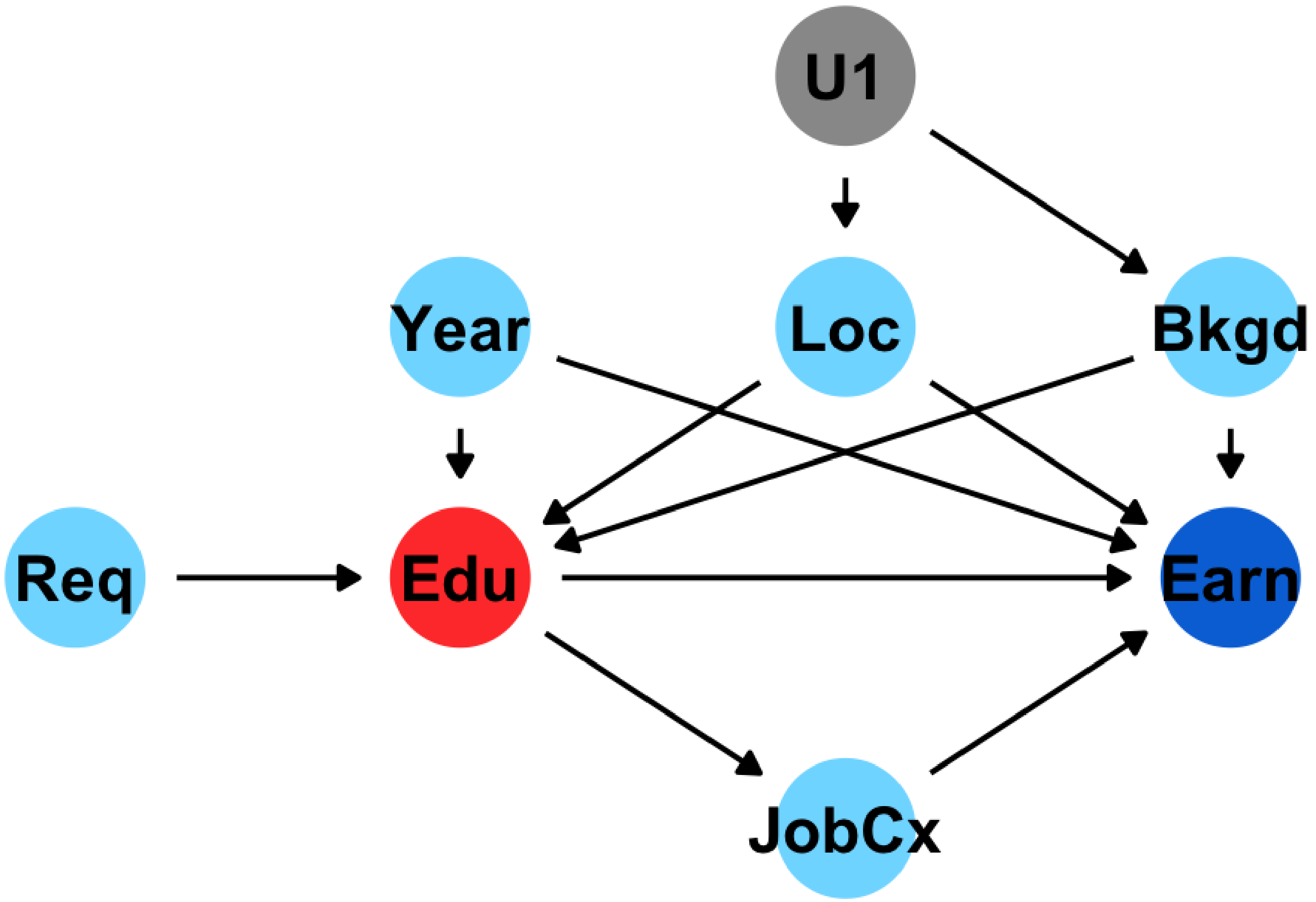
$$
\begin{aligned}
\text{Earn} &= f_{\text{Earn}}(\text{Edu}, \text{Year}, \text{Bkgd}, \\
&\qquad \text{Loc}, \text{JobCx}) \\
\text{Edu} &= f_{\text{Edu}}(\text{Req}, \text{Loc}, \text{Year}) \\
\text{JobCx} &= f_{\text{JobCx}}(\text{Edu}) \\
\text{Bkgd} &= f_{\text{Bkgd}}(\text{U1}) \\
\text{Loc} &= f_{\text{Loc}}(\text{U1})
\end{aligned}
$$

```
dagify(
  Earn ~ Edu + Year + Bkgd + Loc + JobCx,
  Edu ~ Req + Loc + Bkgd + Year,
  JobCx ~ Edu,
  Bkgd ~ U1,
  Loc ~ U1
)
```

Causal identification

All these nodes are related; there's correlation between them all

We care about **Edu** $\longrightarrow$ **Earn**, but what do we do about all the other nodes?



Causal identification

A causal effect is *identified* if the association between treatment and outcome is propertly stripped and isolated

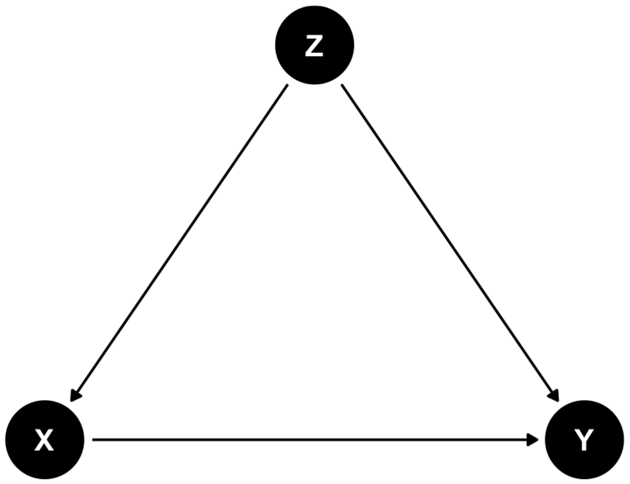
Paths and associations

Arrows in a DAG transmit associations

You can redirect and control those paths by "adjusting" or "conditioning"

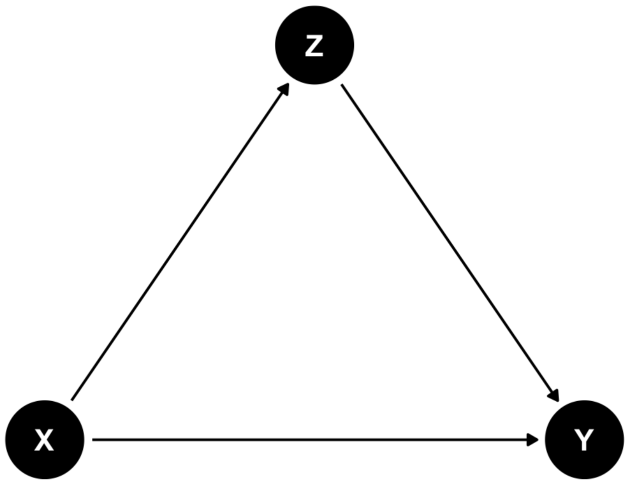
Three types of associations

Confounding



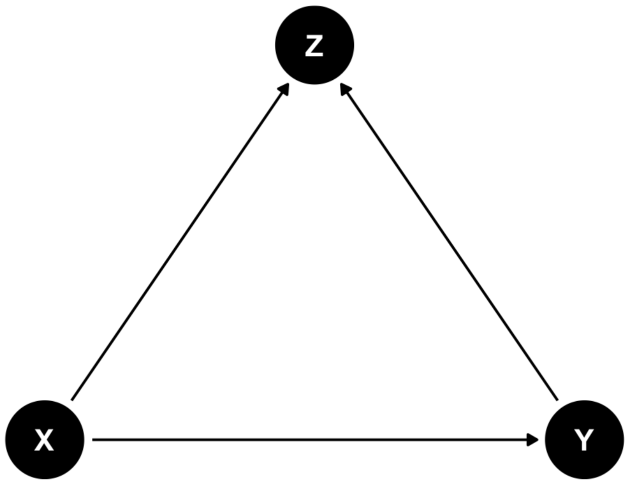
Common cause

Causation



Mediation

Collision



Selection / endogeneity

Interventions

do-operator

Making an intervention in a DAG

$$P[Y \mid do(X = x)] \quad \text{or} \quad E[Y \mid do(X = x)]$$

P = probability distribution, or E = expectation/expected value

Y = outcome, X = treatment;
x = specific value of treatment

Interventions

$$E[Y \mid do(X = x)]$$

E[Earnings | *do*(One year of college)]

E[Firm growth | *do*(Government R&D funding)]

E[Air quality | *do*(Carbon tax)]

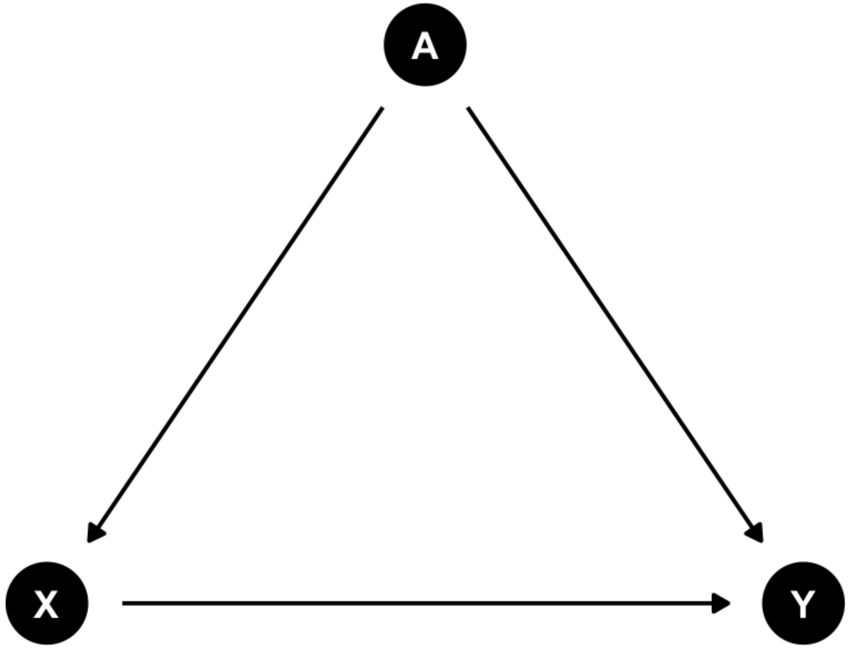
E[Juvenile delinquency | *do*(Truancy program)]

E[Malaria infection rate | *do*(Mosquito net)]

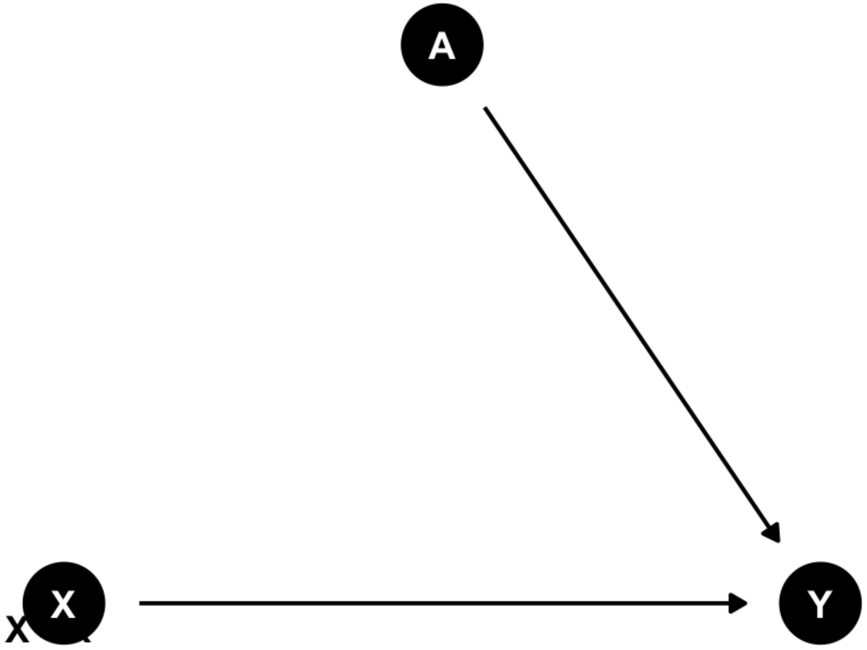
Interventions

When you *do*() X, delete all arrows into it

Observational DAG



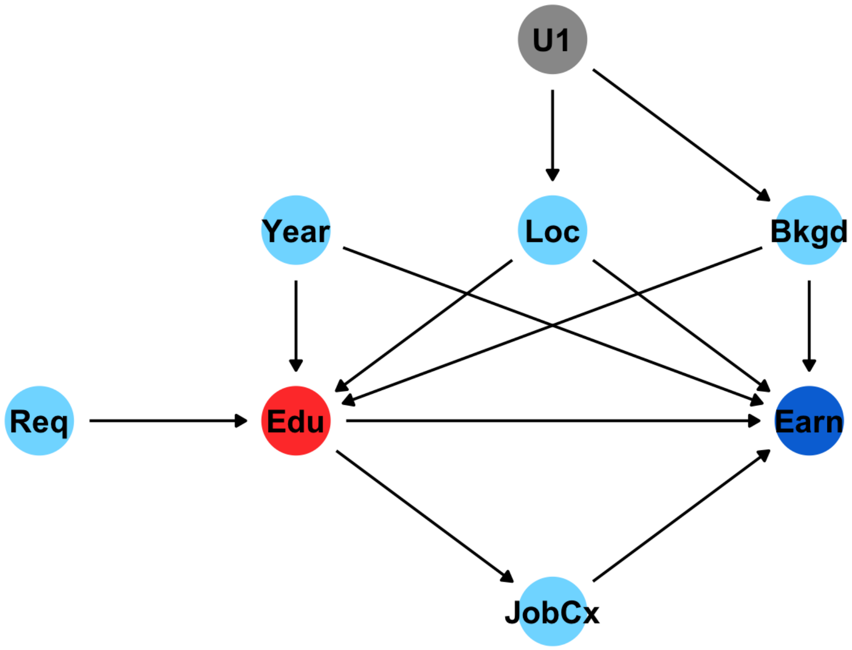
Experimental DAG



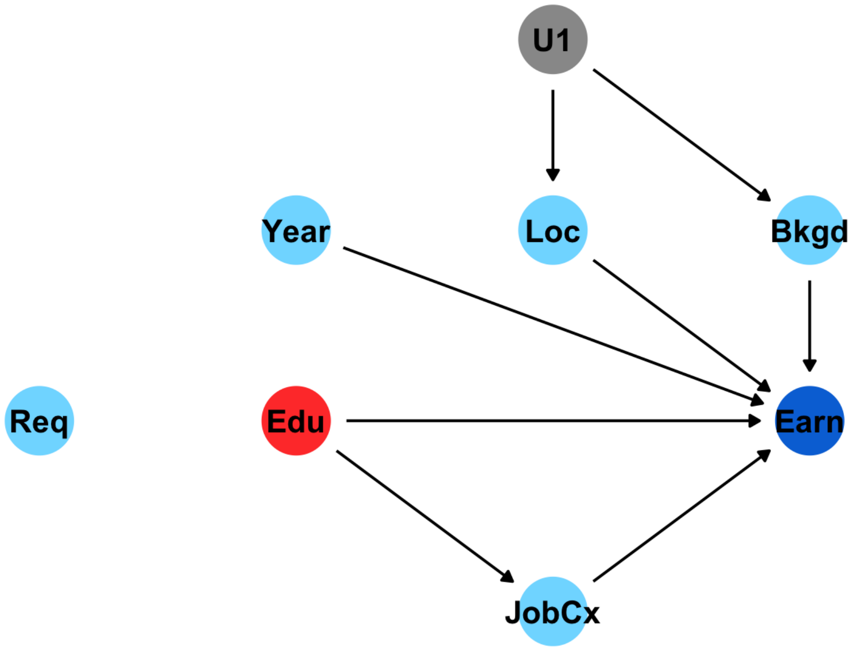
Interventions

$$E[\text{Earnings} \mid do(\text{College education})]$$

Observational DAG



Experimental DAG



Un*do*()ing things

We want to know P[Y | *do*(X)]
but all we have is
observational data X, Y, and Z

$$P[Y \mid do(X)] \neq P(Y \mid X)$$

Correlation isn't causation!

Und*o*()ing things

**Our goal with observational data:
Rewrite P[Y | *do*(X)] so that it doesn't have a *do*() anymore (is "*do*-free")**

do-calculus

A set of three rules that let you manipulate a DAG in special ways to remove *do*() expressions

The do-calculus Let G be a CGM, $G_{\overline{T}}$ represent G post-intervention (i.e with all links into T removed) and $G_{\underline{T}}$ represent G with all links *out of* T removed. Let $do(t)$ represent intervening to set a single variable T to t,

Rule 1: $\mathbb{P}(y|do(t), z, w) = \mathbb{P}(y|do(t), z)$ if $Y \perp\!\!\!\perp W|(Z, T)$ in $G_{\overline{T}}$

Rule 2: $\mathbb{P}(y|do(t), z) = \mathbb{P}(y|t, z)$ if $Y \perp\!\!\!\perp T|Z$ in $G_{\underline{T}}$

Rule 3: $\mathbb{P}(y|do(t), z) = \mathbb{P}(y|z)$ if $Y \perp\!\!\!\perp T|Z$ in $G_{\overline{T}}$, and Z is not a decedent of T.

WAAAAAY beyond the score of this class!
Just know it exists and computer algorithms can do it for you!

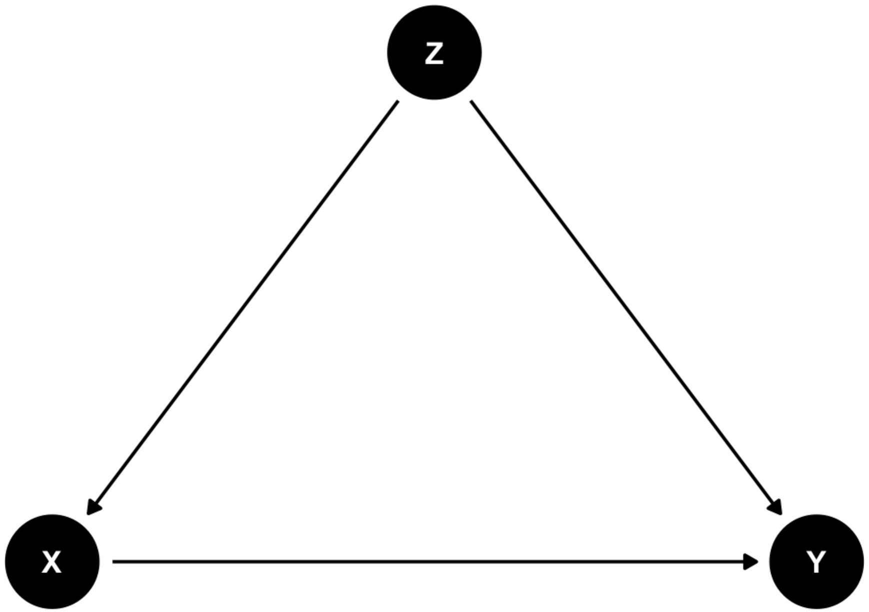
Special cases of *do*-calculus

Backdoor adjustment

Frontdoor adjustment

Backdoor adjustment

$$P[Y \mid do(X)] = \sum_{Z} P(Y \mid X, Z) \times P(Z)$$

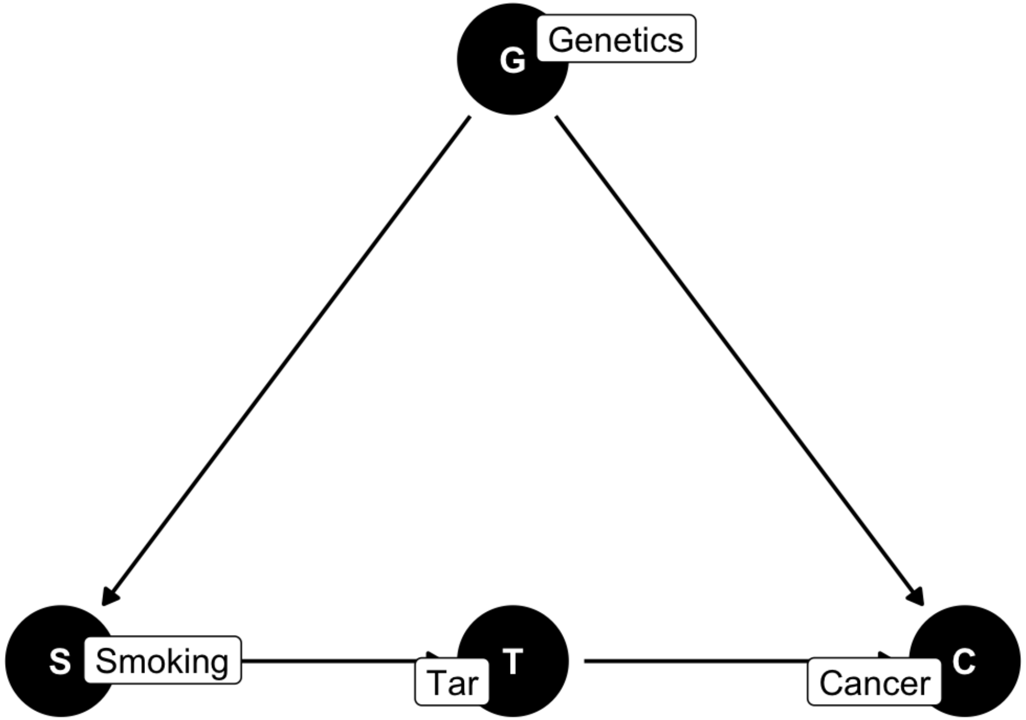


↑ That's complicated!

The right-hand side of the equation means "the effect of X on Y after adjusting for Z"

There's no *do*() on that side!

Frontdoor adjustment



S $\rightarrow$ T is *d*-separated; T $\rightarrow$ C is *d*-separated
combine the effects to find S $\rightarrow$ C

Moral of the story

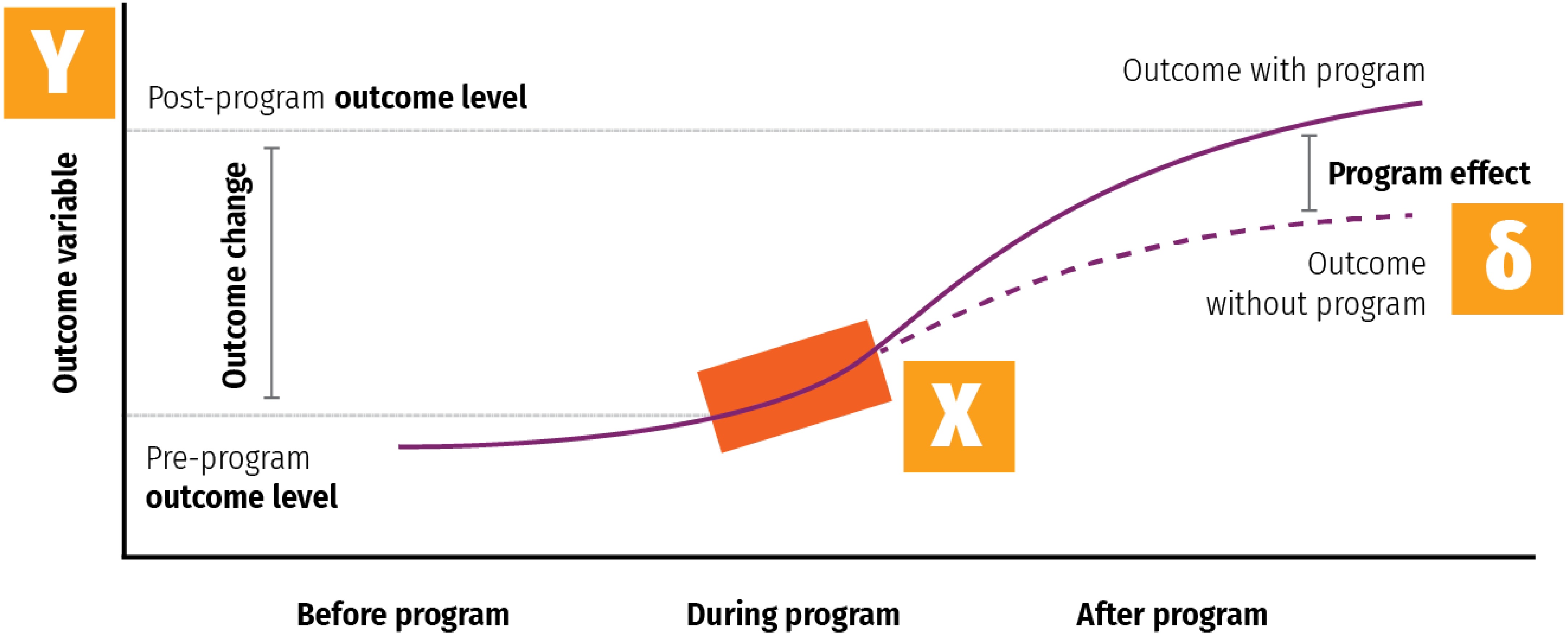
If you can transform *do*() expressions to *do*-free versions, you can legally make causal inferences from observational data

Backdoor adjustment is easiest to see + dagitty and **ggdag** do this for you!

Fancy algorithms (found in the **causaleffect** package) can do the official *do*-calculus for you too

Potential outcomes

Program effect



Some equation translations

Causal effect = δ (delta)

$$\delta = P[Y \mid do(X)]$$

$$\delta = E[Y \mid do(X)] - E[Y \mid \hat{do}(X)]$$

$$\delta = (Y \mid X = 1) - (Y \mid X = 0)$$

$$\delta = Y_1 - Y_0$$

Fundamental problem of causal inference

$$\delta_i = Y_i^1 - Y_i^0 \quad \text{in real life is} \quad \delta_i = Y_i^1 - ???$$

Individual-level effects are impossible to observe!

There are no individual counterfactuals!

Average treatment effect (ATE)

Solution: Use averages instead

$$\text{ATE} = E(Y_1 - Y_0) = E(Y_1) - E(Y_0)$$

Difference between average/expected value when program is on vs. expected value when program is off

$$\delta = (\bar{Y} \mid P = 1) - (\bar{Y} \mid P = 0)$$

Person	Age	Treated	Outcome with program	Outcome without program	Effect
1	Old	TRUE	**80**	60	**20**
2	Old	TRUE	**75**	70	**5**
3	Old	TRUE	**85**	80	**5**
4	Old	FALSE	70	**60**	**10**
5	Young	TRUE	**75**	70	**5**
6	Young	FALSE	80	**80**	**0**
7	Young	FALSE	90	**100**	**-10**
8	Young	FALSE	85	**80**	**5**

Person	Age	Treated	Outcome with program	Outcome without program	Effect
1	Old	TRUE	**80**	60	**20**
2	Old	TRUE	**75**	70	**5**
3	Old	TRUE	**85**	80	**5**
4	Old	FALSE	70	**60**	**10**
5	Young	TRUE	**75**	70	**5**
6	Young	FALSE	80	**80**	**0**
7	Young	FALSE	90	**100**	**-10**
8	Young	FALSE	85	**80**	**5**

$$\delta = (\bar{Y} \mid P = 1) - (\bar{Y} \mid P = 0)$$

$$\text{ATE} = \frac{20 + 5 + 5 + 5 + 10 + 0 + -10 + 5}{8} = 5$$

CATE

ATE in subgroups

Is the program more effective for specific age groups?

Person	Age	Treated	Outcome with program	Outcome without program	Effect
1	Old	TRUE	**80**	60	**20**
2	Old	TRUE	**75**	70	**5**
3	Old	TRUE	**85**	80	**5**
4	Old	FALSE	70	**60**	**10**
5	Young	TRUE	**75**	70	**5**
6	Young	FALSE	80	**80**	**0**
7	Young	FALSE	90	**100**	**-10**
8	Young	FALSE	85	**80**	**5**

$$\delta = (\bar{Y}_\text{O} \mid P = 1) - (\bar{Y}_\text{O} \mid P = 0)$$

$$\delta = (\bar{Y}_\text{Y} \mid P = 1) - (\bar{Y}_\text{Y} \mid P = 0)$$

$$\text{CATE}_\text{Old} = \frac{20 + 5 + 5 + 10}{4} = 10$$

$$\text{CATE}_\text{Young} = \frac{5 + 0 - 10 + 5}{4} = 0$$

ATT and ATU

Average treatment on the treated

ATT / TOT

Effect for those with treatment

Average treatment on the untreated

ATU / TUT

Effect for those without treatment

Person	Age	Treated	Outcome with program	Outcome without program	Effect
1	Old	TRUE	**80**	60	**20**
2	Old	TRUE	**75**	70	**5**
3	Old	TRUE	**85**	80	**5**
4	Old	FALSE	70	**60**	**10**
5	Young	TRUE	**75**	70	**5**
6	Young	FALSE	80	**80**	**0**
7	Young	FALSE	90	**100**	**-10**
8	Young	FALSE	85	**80**	**5**

$$\delta = (\bar{Y}_{\text{T}} \mid P = 1) - (\bar{Y}_{\text{T}} \mid P = 0)$$

$$\delta = (\bar{Y}_{\text{U}} \mid P = 1) - (\bar{Y}_{\text{U}} \mid P = 0)$$

$$\text{CATE}_{\text{Treated}} = \frac{20+5+5+5}{4} = 8.75$$

$$\text{CATE}_{\text{Untreated}} = \frac{10+0-10+5}{4} = 1.25$$

ATE, ATT, and ATU

The ATE is the weighted average of the ATT and ATU

$$\text{ATE} = (\pi_{\text{Treated}} \times \text{ATT}) + (\pi_{\text{Untreated}} \times \text{ATU})$$

$$(\frac{4}{8} \times 8.75) + (\frac{4}{8} \times 1.25)$$

$$4.375 + 0.625 = 5$$

π here means "proportion," not 3.1415

Selection bias

ATE and ATT aren't always the same

ATE = ATT + Selection bias

$$5 = 8.75 + x$$
$$x = -3.75$$

Randomization fixes this, makes x = 0

Actual data

Person	Age	Treated	Actual outcome
1	Old	TRUE	80
2	Old	TRUE	75
3	Old	TRUE	85
4	Old	FALSE	60
5	Young	TRUE	75
6	Young	FALSE	80
7	Young	FALSE	100
8	Young	FALSE	80

Treatment not randomly assigned

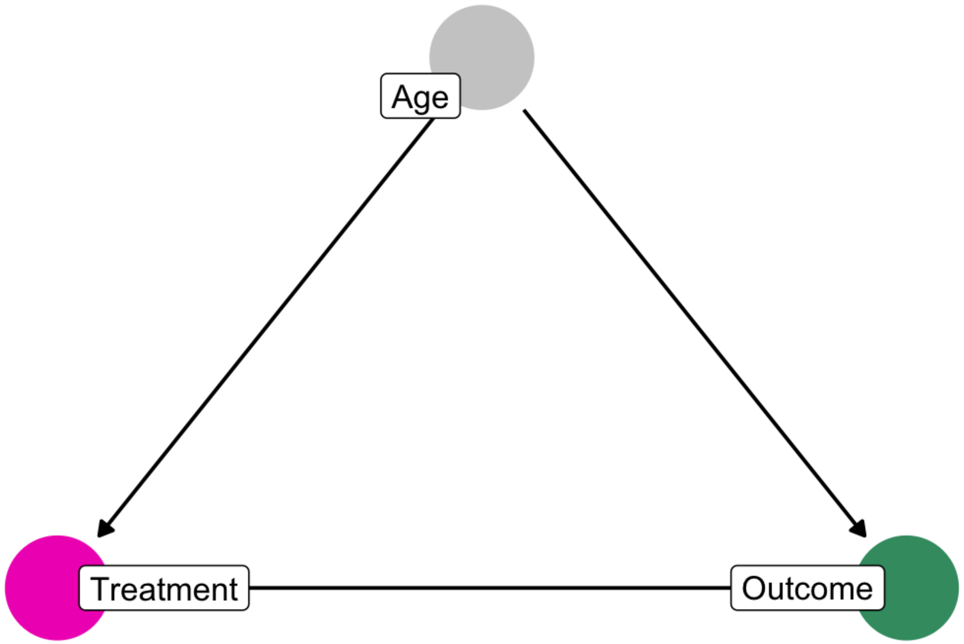
We can't see unit-level causal effects

What do we do?!

Actual data

Person	Age	Treated	Actual outcome
1	Old	TRUE	80
2	Old	TRUE	75
3	Old	TRUE	85
4	Old	FALSE	60
5	Young	TRUE	75
6	Young	FALSE	80
7	Young	FALSE	100
8	Young	FALSE	80

Treatment seems to be correlated with age



Actual data

Person	Age	Treated	Actual outcome
1	Old	TRUE	80
2	Old	TRUE	75
3	Old	TRUE	85
4	Old	FALSE	60
5	Young	TRUE	75
6	Young	FALSE	80
7	Young	FALSE	100
8	Young	FALSE	80

We can estimate the ATE by finding the weighted average of age-based CATEs

As long as we assume/pretend treatment was randomly assigned within each age = unconfoundedness

$$\widehat{\text{ATE}} = \pi_{\text{Old}} \widehat{\text{CATE}}_{\text{Old}} + \pi_{\text{Young}} \widehat{\text{CATE}}_{\text{Young}}$$

Actual data

$$\widehat{\text{ATE}} = \pi_{\text{Old}} \widehat{\text{CATE}}_{\text{Old}} + \pi_{\text{Young}} \widehat{\text{CATE}}_{\text{Young}}$$

Person	Age	Treated	Actual outcome
1	Old	TRUE	80
2	Old	TRUE	75
3	Old	TRUE	85
4	Old	FALSE	60
5	Young	TRUE	75
6	Young	FALSE	80
7	Young	FALSE	100
8	Young	FALSE	80

$$\widehat{\text{CATE}}_{\text{Old}} = \frac{80+75+85}{3} - \frac{60}{1} = 20$$

$$\widehat{\text{CATE}}_{\text{Young}} = \frac{75}{1} - \frac{80+100+80}{3} = -11.667$$

$$\widehat{\text{ATE}} = (\frac{4}{8} \times 20) + (\frac{4}{8} \times -11.667) = 4.1667$$

¡¡¡DON'T DO THIS!!!

$$\widehat{\text{ATE}} = \widehat{\text{CATE}}_{\text{Treated}} - \widehat{\text{CATE}}_{\text{Untreated}}$$

Person	Age	Treated	Actual outcome
1	Old	TRUE	80
2	Old	TRUE	75
3	Old	TRUE	85
4	Old	FALSE	60
5	Young	TRUE	75
6	Young	FALSE	80
7	Young	FALSE	100
8	Young	FALSE	80

$$\widehat{\text{CATE}}_{\text{Treated}} = \frac{80+75+85+75}{4} = 78.75$$

$$\widehat{\text{CATE}}_{\text{Untreated}} = \frac{60+80+100+80}{4} = 80$$

$$\widehat{\text{ATE}} = 78.75 - 80 = -1.25$$

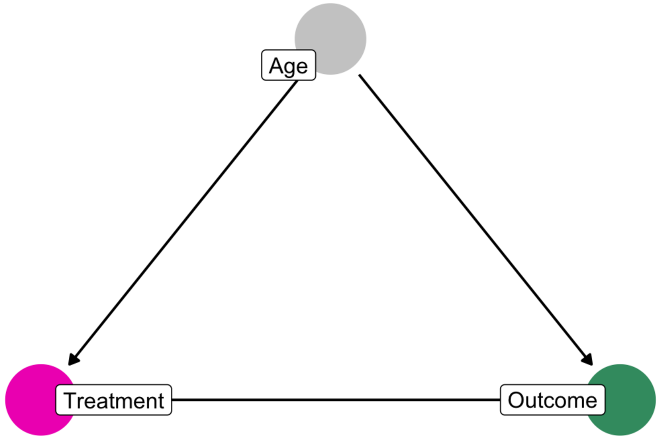
You can only do this if treatment is random!

Matching and ATEs

$$\widehat{\text{ATE}} = \pi_{\text{Old}} \widehat{\text{CATE}}_{\text{Old}} + \pi_{\text{Young}} \widehat{\text{CATE}}_{\text{Young}}$$

We used age here because it correlates with (and confounds) the outcome

And we assumed unconfoundedness; that treatment is randomly assigned within the groups



Does attending a private university cause an increase in earnings?

TABLE 2.1
The college matching matrix

		Private			Public			
Applicant group	Student	Ivy	Leafy	Smart	All State	Tall State	Altered State	1996 earnings
A	1		Reject	**Admit**		Admit		110,000
	2		Reject	**Admit**		Admit		100,000
	3		Reject	Admit		**Admit**		110,000
B	4	**Admit**			Admit		Admit	60,000
	5	Admit			Admit		**Admit**	30,000
C	6		**Admit**					115,000
	7		**Admit**					75,000
D	8	Reject			**Admit**	Admit		90,000
	9	Reject			Admit	**Admit**		60,000

Note: Enrollment decisions are highlighted in gray.

Table 2.1
The college matching matrix

Applicant group	Student	Private: Ivy	Private: Leafy	Private: Smart	Public: All State	Public: Tall State	Public: Altered State	1996 earnings
A	1		Reject	Admit		Admit		110,000
	2		Reject	Admit		Admit		100,000
	3		Reject	Admit		Admit		110,000
B	4	Admit			Admit		Admit	60,000
	5	Admit			Admit		Admit	30,000
C	6		Admit					115,000
	7		Admit					75,000
D	8	Reject			Admit	Admit		90,000
	9	Reject			Admit	Admit		60,000

Note: Enrollment decisions are highlighted in gray.

This is tempting!

Average private – Average public

$$\frac{110 + 100 + 60 + 115 + 75}{5} = 92$$

$$\frac{110 + 30 + 90 + 60}{4} = 72.5$$

$$\left(92 \times \frac{5}{9}\right) - \left(72.5 \times \frac{4}{9}\right) = 18,888$$

This is wrong!

$$\widehat{\text{ATE}} = \pi_{\text{Private}} \widehat{\text{CATE}}_{\text{Private}} + \pi_{\text{Public}} \widehat{\text{CATE}}_{\text{Public}}$$

Grouping and matching

Table 2.1
The college matching matrix

		Private			Public			
Applicant group	Student	Ivy	Leafy	Smart	All State	Tall State	Altered State	1996 earnings
A	1		Reject	Admit		Admit		110,000
	2		Reject	Admit		Admit		100,000
	3		Reject	Admit		Admit		110,000
B	4	Admit			Admit		Admit	60,000
	5	Admit			Admit		Admit	30,000
C	6		Admit					115,000
	7		Admit					75,000
D	8	Reject			Admit	Admit		90,000
	9	Reject			Admit	Admit		60,000

Note: Enrollment decisions are highlighted in gray.

These groups look like they have similar characteristics

Unconfoundedness?

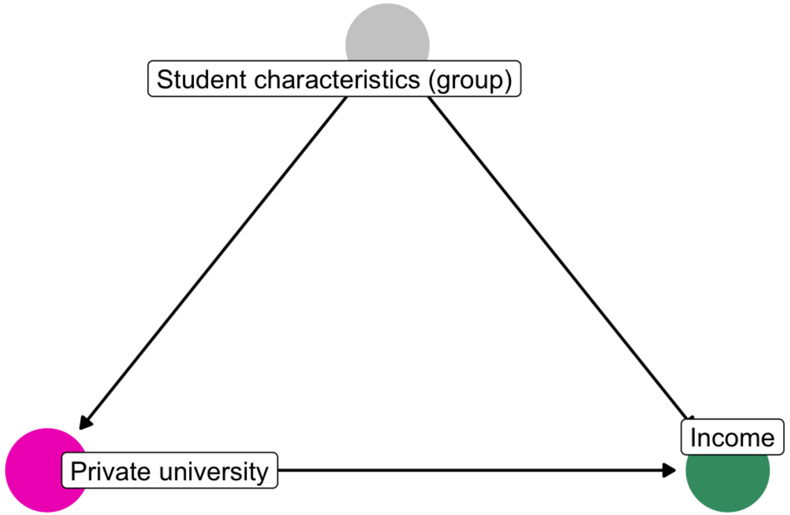


TABLE 2.1
The college matching matrix

Applicant group	Student	Private: Ivy	Private: Leafy	Private: Smart	Public: All State	Public: Tall State	Public: Altered State	1996 earnings
A	1		Reject	Admit		Admit		110,000
	2		Reject	Admit		Admit		100,000
	3		Reject	Admit		Admit		110,000
B	4	Admit			Admit		Admit	60,000
	5	Admit			Admit		Admit	30,000
C	6		Admit					115,000
	7		Admit					75,000
D	8	Reject			Admit	Admit		90,000
	9	Reject			Admit	Admit		60,000

Note: Enrollment decisions are highlighted in gray.

CATE Group A + CATE Group B

$$\frac{110 + 100}{2} - 110 = -5,000$$

$$60 - 30 = 30,000$$

$$(-5 \times \frac{3}{5}) + (30 \times \frac{2}{5}) = 9,000$$

This is less wrong!

$$\widehat{\text{ATE}} = \pi_{\text{Group A}} \widehat{\text{CATE}}_{\text{Group A}} + \pi_{\text{Group B}} \widehat{\text{CATE}}_{\text{Group B}}$$

Matching with regression

$$\text{Earnings} = \alpha + \beta_1 \text{Private} + \beta_2 \text{Group} + \epsilon$$

```
model_earnings <- lm(earnings ~ private + group_A, data = schools_small)
```

term	estimate	std.error	statistic	p.value
(Intercept)	40000	11952.29	3.35	0.08
privateTRUE	10000	13093.07	0.76	0.52
group_ATRUE	60000	13093.07	4.58	0.04

β_1 = $10,000

This is less wrong!

Significance details!